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Option pricing by large risk aversion utility under transaction costs

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Abstract. We consider a multi-asset continuous-time model of a financial market with transaction costs and prove that, for a strongly risk-averse investor, the reservation price of a contingent claim approaches the super-replication price increased by the liquidation value of the initial endowment.

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1. Introduction

For frictionless markets the pricing of contingent claims can be based on the following simple idea suggested in Hodges and Neuberger (1989). Let $x \geq 0$ be the endowment of an agent at time zero and let $\mathcal{X}(x)$ be the set of terminal values of feasible portfolios at the time horizon T; typically, $\mathcal{X}(x) = x + \mathcal{X}(0)$. Let U be a utility function, e.g., $U(r) = 1 - e^{-\eta r}$, where large values of the positive parameter η mean that the agent is strongly risk-averse. Selling an option for the price h at time zero, the agent starts with capital $x + h$ but his results at the date T will be diminished by a liability G. The deal is attractive to the seller if

$$\sup_{\xi \in \mathcal{X}(x+h)} EU(\xi - G) > \sup_{\xi \in \mathcal{X}(x)} EU(\xi).$$

The infimum of such h is the *reservation price* of the option; in general, it depends on x.

The utility-based approach to option pricing attracted attention because for many models the super-replication price is too high. The modern trend is to study this problem in the context of more realistic models with transaction costs (see some relevant references at the end of the paper). The main difficulty of the theory with friction is that not only the price process but also other basic objects are vector quantities (e.g., value processes, contingent claims, etc.); this requires an intensive use of methods of convex geometry.

Working with a rather general multi-asset model suggested in Kabanov and Last (1999), we show that, if the seller is strongly risk-averse, his reservation price approaches the super-replication price increased by the liquidation value of the initial endowment. In the special case where the initial holdings in risky assets are zero, this result was originally conjectured by Barles and Soner (1998). In contrast to Bouchard (2000), where the result was obtained in a Markovian setting via an asymptotic analysis of viscosity solutions of HJB equations, we use much easier and less restrictive duality methods which also provide a better insight into the essence of the problem. Our main theorem can be compared with the corresponding result on risk-averse asymptotics for frictionless markets in the recent work by Delbaen et al. (2000).

2. Problem formulation

2.1. Financial market with transaction costs

Let T be a finite time horizon and let $(\Omega, \mathcal{F}, \mathbb{F} = (\mathcal{F}_t)_{t\le T}, P)$ be a stochastic basis with the trivial σ-algebra $\mathcal{F}_0$. Let $S := (S^1, \dots, S^d)$ be a semimartingale with strictly positive components and $S_0^i = 1$; the first component is assumed to be constant over time. With the interpretation of S as a price process, this means that the first security ("cash") is taken as the numéraire.

A *trading strategy* is an adapted, right-continuous, (componentwise) non-decreasing process L taking values in $\mathbb{M}_+^d$, the set of $d \times d$-matrices with non-negative entries; L_t^{ij} is the cumulative net amount of funds transferred from the asset i to the asset j up to the date t; this process may have a jump at the origin $\Delta L_0^{ij} = L_0^{ij}$ corresponding to the initial transfer. Constant proportional transaction costs are described by a matrix $(\lambda^{ij}) \in \mathbb{M}_+^d$ with zero diagonal. Without loss of generality, we can assume that

$$(1+\lambda^{ij})(1+\lambda^{jk}) \le (1+\lambda^{ik}) \quad \text{for all } i, j, k = 1, \dots, d. \tag{2.1}$$

This means that the transaction costs implied by a transfer from asset i to k can not be reduced by an artificial transit through an asset j. Condition (2.1) above is only used to obtain an explicit characterization of Property $\mathbf{H}_3$ in Section 3 below.

For an initial holdings vector $x \in \mathbb{R}^d$ and a strategy L, the portfolio holdings $X = X^{x,L}$ are defined by the dynamics

$$X_t^i = x + \int_0^t \widehat{X}_{r-}^i \, dS_r^i + \sum_{j=1}^d \left(L_t^{ji} - (1 + \lambda^{ij}) L_t^{ij} \right)$$

where $\widehat{X}^i := X^i / S^i$ (i.e., $\widehat{X}$ is the process X divided by the process S componentwise).

The *solvency region* is the following convex polyhedral cone in $\mathbb{R}^d$:

$$K := \left\{ x : \exists a \in \mathbb{M}_+^d \text{ such that } x^i + \sum_{j=1}^d (a^{ji} - (1 + \lambda^{ij}) a^{ij}) \geq 0, \ i \leq d \right\}.$$

It generates a partial ordering: $x \succeq y$ if $x - y \in K$.

The set $K^* := \left\{ w \in \mathbb{R}^d : \ wx \geq 0 \, \forall \, x \in K \right\}$ is the positive polar cone of K. The partial ordering can be characterized in terms of K^*: $x \succeq 0$ iff $wx \geq 0$ for all $w \in K^*$. Direct computation shows that

$$K^* = \left\{ w \in \mathbb{R}^d : \ w^j - (1 + \lambda^{ij}) w^i \leq 0, \ 1 \leq i, j \leq d \right\}. \tag{2.2}$$

A trading strategy L is *admissible* if there is a constant $c_L \geq 0$ such that

$$X_t^{x,L} \succeq -c_L S_t, \quad t \leq T. \tag{2.3}$$

It is easy to check that the set of admissible strategies $\mathcal{A}$ does not depend on x. We denote by $\mathcal{X}(x)$ the convex set of terminal values of portfolio processes starting from the initial endowment x, i.e.,

$$\mathcal{X}(x) := \{ X_T^{x,L} : \ L \in \mathcal{A} \}.$$

Finally, $\mathbf{1}_1 := (1, 0, \ldots, 0)$, $\mathbf{1} := (1, \ldots, 1)$, and c will denote a real constant.

Remark 2.1. Let $x \in \mathbb{R}^d$ and $L \in \mathcal{A}$. Then $X^{x,L} + c\mathbf{1}_1 = X^{x + c\mathbf{1}_1, L}$ and

$$\mathcal{X}(x + c\mathbf{1}_1) = \mathcal{X}(x) + c\mathbf{1}_1.$$

These relations hold because the first asset is the numéraire.

2.2. *Reservation price*

As in Bouchard (2000), we define the *liquidation value*

$$\ell(x) := \sup\{r \in \mathbb{R} : \ x \succeq r\mathbf{1}_1\},$$

the maximal cash endowment that one can get from x by clearing all the positions in the risky assets. Easily verified properties of the liquidation function ℓ are summarized in

Proposition 2.1. *Let $x, y \in \mathbb{R}^d$. Then*

(a) $x \succeq \ell(x)\mathbf{1}_1$ *(i.e., the* sup *is attained);*
(b) $\ell(x+y) \geq \ell(x) + \ell(y)$*;*
(c) $\ell(cx) = c\ell(x)$ *for* $c \geq 0$*;*
(d) $\ell(x) \geq 0$ *if* $x \succeq 0$*;*
(e) $\ell(x + c\mathbf{1}_1) = \ell(x) + c$*;*
(f) $\ell(x) = \inf\{wx : \ w \in K^*, \ w^1 = 1\}$.

Remark 2.2. It follows from (a) that $\ell(x)\mathbf{1}_1 \in \mathcal{X}(x)$.

We fix a *contingent claim* G, a d-dimensional $\mathcal{F}_T$-measurable random variable, assuming that $G \succeq -cS_T$ for some $c \in \mathbb{R}$.

Let $\eta > 0$ and

$$U_\eta(r) := -e^{-\eta r}, \quad r \in \mathbb{R}.$$

Similarly to Hodges and Neuberger (1989), we suppose that the agent's decision to sell the contingent claim is based on a comparison of expected utilities of the initial endowments in two portfolio optimization problems:

$$V_\eta^0(x) := \sup_{X \in \mathcal{X}(x)} EU_\eta\left(\ell(X)\right)$$

(the contingent claim is sold) and

$$V_\eta^G(x) := \sup_{X \in \mathcal{X}(x)} EU_\eta\left(\ell(X - G)\right)$$

(the contingent claim is not sold).

The *reservation price* (for the seller)

$$p_\eta(x) := \inf\left\{r \in \mathbb{R} : \ V_\eta^G(x + r\mathbf{1}_1) \geq V_\eta^0(x)\right\}$$

is the minimal initial cash endowment which induces a higher maximal expected utility with the "liability" G at the terminal date T.

The particular form of the exponential utility function implies that

$$p_\eta(x) = \frac{1}{\eta} \ln \frac{V_\eta^G(x)}{V_\eta^0(x)}, \tag{2.4}$$

and this identity follows from a slightly more general assertion.

Lemma 2.1. *Let*

$$p_\eta(x,y) := \inf\left\{r \in \mathbb{R} :\ V_\eta^G(x + r\mathbf{1}_1) \geq V_\eta^0(y)\right\}.$$

Then

$$p_\eta(x,y) = \frac{1}{\eta}\ln\frac{V_\eta^G(x)}{V_\eta^0(y)}.$$

Proof. Set $p := p_\eta(x,y)$. The function $V_\eta^G(x)$ is increasing and continuous in the x^1 variable. Thus, $V_\eta^0(y) = V_\eta^G(x_1 + p\mathbf{1}_1)$. By Remark 2.1

$$V_\eta^G(x + p\mathbf{1}_1) = V_\eta^{G-p\mathbf{1}_1}(x) = e^{-\eta p}V_\eta^G(x)$$

and the result follows. $\square$

Finally, we introduce the set of hedging endowments

$$\Gamma := \{x \in \mathbb{R}^d :\ X \succeq G \text{ for some } X \in \mathcal{X}(x)\},$$

and the *super-replication price* of the contingent claim

$$g(x) := \inf\{r \in \mathbb{R} :\ x + r\mathbf{1}_1 \in \Gamma\},$$

i.e., the minimal initial cash increase needed in order to hedge G without risk, starting with the initial endowment x.

Lemma 2.2. *Let $x, y \in \mathbb{R}^d$. Then*

$$\frac{1}{\eta}\ln[-V_\eta^G(x)] - g(x) + \ell(-y) \ \leq\ \frac{1}{\eta}\ln[-V_\eta^0(y] \ \leq\ -\ell(y).$$

Proof. The second inequality is obvious: $\ell(y)\mathbf{1}_1 \in \mathcal{X}(y)$ (Remark 2.2) and hence $V_\eta^0(y) \geq -e^{-\eta\ell(y)}$. By Lemma 2.1 the first inequality means that $p_\eta(x,y) \leq g(x) - \ell(-y)$. To prove this bound in the non-trivial case where $g(x) < \infty$ we take $\varepsilon > 0$ and $\xi \in \mathcal{X}(y)$. By definition of $g(x)$, there is $\tilde{\xi} \in \mathcal{X}(x + (g(x) + \varepsilon)\mathbf{1}_1)$ such that $\tilde{\xi} - G \succeq 0$. Since $-\ell(-y)\mathbf{1}_1 \succeq y$, there is $\xi' \in \mathcal{X}(-\ell(-y)\mathbf{1}_1)$ such that $\xi' \succeq \xi$. Put $r_\varepsilon := -\ell(-y) + g(x) + \varepsilon$ and $\xi'' := \tilde{\xi} + \xi'$. Then $\xi'' \in \mathcal{X}(x + r_\varepsilon\mathbf{1}_1)$ and $\xi'' - G \succeq \xi$; hence, $\ell(\xi'' - G) \geq \ell(\xi)$. Since $\xi \in \mathcal{X}(y)$ is arbitrary, we obtain the bound $V_\eta^G(x + r_\varepsilon\mathbf{1}_1) \geq V_\eta^0(y)$ implying that $p_\eta(x,y) \leq r_\varepsilon$. $\square$

3. The main result

Let $\mathcal{M}$ denote the space of martingales and let

$$\mathcal{D} := \{Z \in \mathcal{M} : \ Z_0^1 = 1, \ \widehat{Z}_t \in K^* \ \forall \, t \le T\},$$
$$\mathcal{D}_e := \{Z \in \mathcal{D} : \ Z_T^1 > 0, \ Z_T^1 \ln Z_T^1 \in L^1\}.$$

The set $\mathcal{D}^T$ of terminal values of processes from $\mathcal{D}$ plays a similar role to that of the set of absolute continuous martingale measures in the theory of frictionless markets while $\mathcal{D}_e^T$ corresponds to the set of equivalent martingale measures with finite entropy. By virtue of (2.2) the elements of $\mathcal{D}_e$ have all components strictly positive. Recall that (2.3) ensures that, for every admissible strategy L, the process $\widehat{Z}X^{x,L}$ is a supermartingale and $E\widehat{Z}_T X_T^{x,L} \le \widehat{Z}_0 x$.

We introduce the following hypotheses.

$\mathbf{H_1}$. The process S is continuous.
$\mathbf{H_2}$. There exists a probability measure $Q \sim P$ such that $S \in \mathcal{M}(Q)$.
$\mathbf{H_3}$. The cone K is proper (i.e., $K \cap (-K) = \{0\}$).
$\mathbf{H_4}$. The set $\mathcal{D}_e \neq \emptyset$ and $\mathcal{D}_e^T$ is dense in $\mathcal{D}^T$ in L^1.

The third condition has various equivalent forms: the interior K^* is non-empty, or (under Condition (2.1)) $\lambda^{ij} + \lambda^{ji} > 0$ for any $i, j, i \neq j$.

Our main result is

Theorem 3.1. *Assume that* $\mathbf{H_1}$–$\mathbf{H_4}$ *hold. Then*

$$\lim_{\eta \to \infty} p_\eta(x) = g(x) + \ell(x).$$

Remark 3.1. Consider the special case $x = a\mathbf{1}_1$ for some $a \in \mathbb{R}$, i.e., the initial holdings in risky assets are zero. Then, from the obvious identity

$$g(a\mathbf{1}_1) + \ell(a\mathbf{1}_1) = g(0),$$

Theorem 3.1 says that the reservation price of the strongly risk averse seller approaches the super-replication cost, as was conjectured in Barles and Soner (1998).

Theorem 3.1 is a simple corollary of the hedging theorem and the following more technical assertion which we prove in Section 4.

Theorem 3.2. *Assume that* $\mathbf{H_2}$ *and* $\mathbf{H_4}$ *hold. Then*

$$\limsup_{\eta \to \infty} p_\eta(x) \le g(x) + \ell(x),$$
$$\liminf_{\eta \to \infty} p_\eta(x) \ge \gamma(x) + \ell(x),$$

where

$$\gamma(x) := \sup_{Z \in \mathcal{D}} E(\widehat{Z}_T G - \widehat{Z}_0 x).$$

Proof of Theorem 3.1. In view of the above inequalities it remains to check that $g(x) = \gamma(x)$ when $\gamma(x) < \infty$. According to Kabanov and Last (1999), if $\mathbf{H_1}$–$\mathbf{H_3}$ hold, then $\Gamma = \{x : \gamma(x) \le 0\}$. In particular, Γ is closed and, hence, $x + g(x)\mathbf{1}_1 \in \Gamma$. For any real c we have $\gamma(x + c\mathbf{1}_1) = \gamma(x) - c$. Therefore,

$$0 \ge \gamma(x + g(x)\mathbf{1}_1) \;=\; \gamma(x) - g(x).$$

On the other hand, since $\gamma\,(x + \gamma(x)\mathbf{1}_1) = 0$, the point $x + \gamma(x)\mathbf{1}_1$ is in Γ and $\gamma(x) \ge g(x)$. $\square$

4. Proof of Theorem 3.2

Lemma 4.1. *For arbitrary $Z \in \mathcal{D}_e$ the following inequality holds:*

$$\frac{1}{\eta}\ln[-V_\eta^G(x)] \ge E(\widehat{Z}_T G - \widehat{Z}_0 x) - \frac{1}{\eta} E Z_T^1 \ln Z_T^1.$$

Proof. Let $\xi \in \mathcal{X}(x)$. Since $\widehat{Z}_T \in K^*$ and $\widehat{Z}_T^1 = Z_T^1 > 0$ we have by Proposition 2.1 (f) that

$$\ell(\xi - G) \le \frac{\widehat{Z}_T}{Z_T^1}(\xi - G).$$

Thus,

$$\begin{aligned}
EU_\eta(\ell(\xi - G)) &\le EU_\eta\left(\frac{\widehat{Z}_T\xi - \widehat{Z}_T G}{Z_T^1}\right)\\
&= E Z_T^1 U_\eta\left(\frac{\widehat{Z}_T\xi - \widehat{Z}_T G}{Z_T^1} + \frac{1}{\eta}\ln Z_T^1\right)\\
&\le U_\eta\left(E(\widehat{Z}_T\xi - \widehat{Z}_T G) + \frac{1}{\eta} E Z_T^1 \ln Z_T^1\right)
\end{aligned}$$

by the Jensen inequality applied with the measure $P^1 := Z_T^1 P$. Since $E\widehat{Z}_T\xi \le \widehat{Z}_0 x$, it follows that

$$V_\eta^G(x) \le U_\eta\left(E(\widehat{Z}_0 x - \widehat{Z}_T G) + \frac{1}{\eta} E Z_T^1 \ln Z_T^1\right).$$

Obviously, this bound is equivalent to the assertion of the lemma. $\square$

Lemma 4.2. *Under* $\mathbf{H_2}$ *and* $\mathbf{H_4}$

$$\begin{aligned}
\ell(x) &= \inf_{Z \in \mathcal{D}_e} \widehat{Z}_0 x,\\
\gamma(x) &= \sup_{Z \in \mathcal{D}_e} E(\widehat{Z}_T G - \widehat{Z}_0 x).
\end{aligned}$$

Proof. For any $w \in K^*$ the process Z with $\widehat{Z}_t = (dQ_t/dP_t)w$ is in $\mathcal{D}$. Thus, $\mathbf{H_2}$ ensures that $K^* \cap \{w : \; w^1 = 1\}$ coincides with the set $\{\widehat{Z}_0 : \; Z \in \mathcal{D}\}$. But by $\mathbf{H_4}$ the set $\{\widehat{Z}_0 : \; Z \in \mathcal{D}_e\}$ is dense in the latter (if the terminal values of martingales converge in L^1 then the initial values also converge). The first identity now follows from Proposition 2.1 (f).

Let $Z \in \mathcal{D}$ and $Z^n \in \mathcal{D}_e$ be such that Z^n_T converges to Z_T in the L^1-sense and also a.s. Since $G \succeq -cS_T$ we have the bound

$$\widehat{Z}^n_T G \geq -c\widehat{Z}^n_T S_T = -c\sum_{i=1}^{d} Z^{ni}_T .$$

Hence, by Fatou's Lemma,

$$E\widehat{Z}_T G - \widehat{Z}_T x \leq \liminf_n (E\widehat{Z}^n_T G - \widehat{Z}^n_0 x) \leq \sup_{Z\in\mathcal{D}_e} E(\widehat{Z}_T G - \widehat{Z}_0 x)$$

and the second identity holds. □

Now the proof of Theorem 3.2 is easy. First we check that

$$\lim_{\eta\to\infty} \frac{1}{\eta} \ln[-V^0_\eta(x)] = -\ell(x). \tag{4.1}$$

Indeed, applying Lemma 4.1 with $G = 0$, we see that

$$\liminf_{\eta\to\infty} \frac{1}{\eta} \ln[-V^0_\eta(x)] \geq -\inf_{Z\in\mathcal{D}_e} \widehat{Z}_0 x \;=\; -\ell(x)$$

by virtue of Proposition 2.1 (f) and the hypothesis $\mathbf{H_4}$. The converse inequality follows from the upper bound of Lemma 2.2.

As a corollary of (4.1), we have that

$$\lim_{\eta\to\infty} \frac{1}{\eta} \ln[-V^0_\eta(x)] = \lim_{\eta\to\infty} \frac{1}{\eta} \ln[-V^0_\eta(\ell(x)\mathbf{1}_1)].$$

It follows from (2.4) and this identity that

$$\begin{aligned}\limsup_{\eta\to\infty} p_\eta(x) &= \limsup_{\eta\to\infty} \frac{1}{\eta}\Big(\ln[-V^G_\eta(x)] - \ln[-V^0_\eta(\ell(x)\mathbf{1}_1)]\Big)\\ &\leq g(x) - \ell(-\ell(x)\mathbf{1}_1) = g(x) + \ell(x),\end{aligned}$$

where we used the lower bound of Lemma 2.2 with $y = \ell(x)\mathbf{1}_1$.

Finally, by virtue of Lemmata 4.1 and 4.2

$$\liminf_{\eta\to\infty} \frac{1}{\eta} \ln[-V^G_\eta(x)] \geq \sup_{Z\in\mathcal{D}_e} E(\widehat{Z}_T G - \widehat{Z}_0 x) = \gamma(x).$$

The second inequality of Theorem 3.2 follows from (2.4) and (4.1). □

5. Final comments

An inspection of the proof shows that in Theorem 3.1 the hypotheses $\mathbf{H_1}$–$\mathbf{H_3}$ can be replaced by a single hypothesis: the closure of Γ coincides with the set $\{x : \gamma(x) \leq 0\}$. Thus, the assertion holds for a discrete-time model (where all strategies are admissible) under $\mathbf{H_2}$ and $\mathbf{H_4}$ alone. Indeed, the hedging theorem in Delbaen et al. (1999) ensures that $\Gamma = \{x : \gamma(x) \leq 0\}$ if $\mathbf{H_2}$ holds. One may expect that the hedging theorems are true under weaker assumptions which automatically will substitute for $\mathbf{H_1}$–$\mathbf{H_3}$ in our formulation.

The large risk-averse asymptotics in Delbaen et al. (2000) are obtained by assuming that all martingales on the stochastic basis are continuous. In fact, one needs only the property that the set $\mathcal{Q}_e$ of martingale measures with finite entropy is dense in the set of all local martingale measures $\mathcal{Q}$. It was shown in Kabanov and Stricker (2000) that this is always the case if $\mathcal{Q}_e \neq \emptyset$. The corresponding result for the sets $\mathcal{D}$ and $\mathcal{D}_e$ is not available yet but, if all martingales on stochastic bases are continuous and $\mathcal{Q}_e \neq \emptyset$, $\mathbf{H_4}$ holds; see the proof of Theorem 3.7 in Kabanov (1999).

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