Introduction to electricity risks, markets and trading

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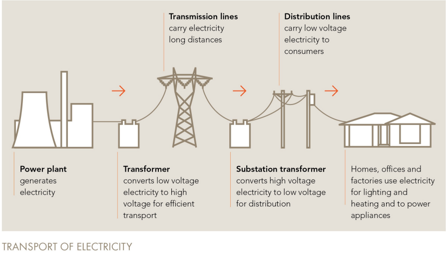
Overview of the course

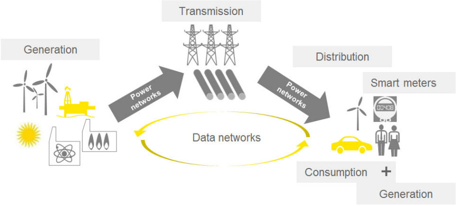
1. **The power generation sector; electricity as a commodity**
   Electricity sector in the world, energy transition. Features of electricity, properties of electricity demand, transport networks. Functioning of electric systems and role of system operators. Network stability and frequency control. Specific risks associated to intermittent renewable electricity generation.
2. **Electricity markets and electricity derivatives**
   Ways to sell electricity. Organization of electricity markets: balancing; intraday, day-ahear, forward and capacity markets. Electricity futures and other derivative products.
3. **Modeling electricity prices and trading in electricity markets**
   Stochastic models for intraday, spot and forward prices. Trading strategies for power producers.

Outline

The electricity supply chain



Source: `eex.gov.au`



Source: `ey.com`

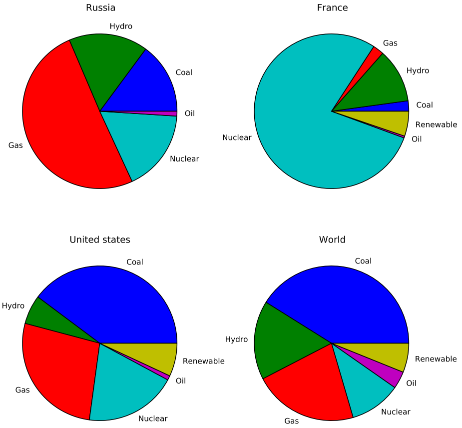
Historically, energy production and distribution was carried out by integrated energy companies; electricity was produced by steam turbines at large power plants and sold to consumers at regulated tariffs

Nowadays, electricity production is competitive, but transport and distribution continue to be managed by state companies; distributed renewable generation plays an important role and data networks and smart meters are used for load management

The electricity sector

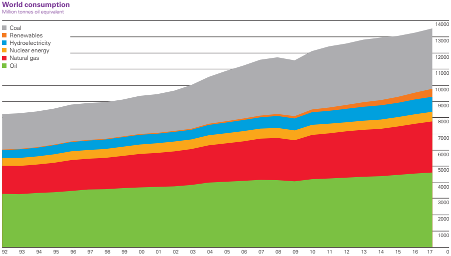
- In most countries the electricity sector is competitive, generation is separated from transport and distribution.
- The transport (high voltage grid) is a natural monopoly and usually managed by Transportation Service Operator (TSO), a state company (in Russia, Федеральная Сетевая Компания, in France RTE), who is also responsible for real time operation and equilibrium of the energy system.
- The distribution and generation companies are independent competing entities from public or private sector.

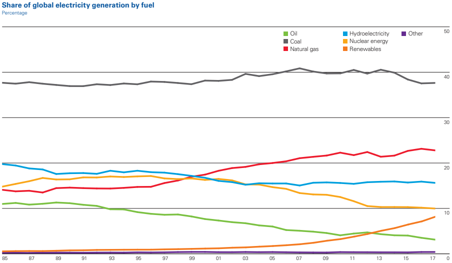
The energy mix



Energy mix in selected countries and the world

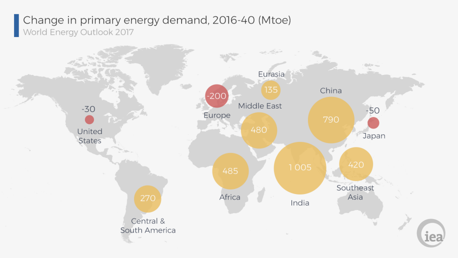
The energy mix

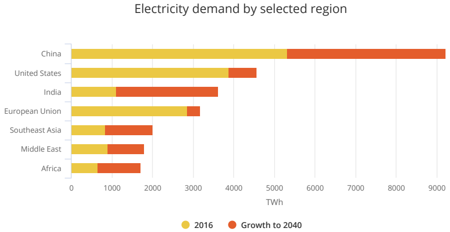


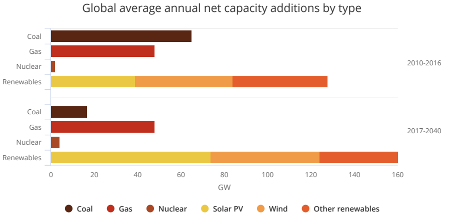


Left: global energy consumption. Right: global electricity generation. Source: BP Statistical Report on World Energy 2018.

Future evolution of the world energy production

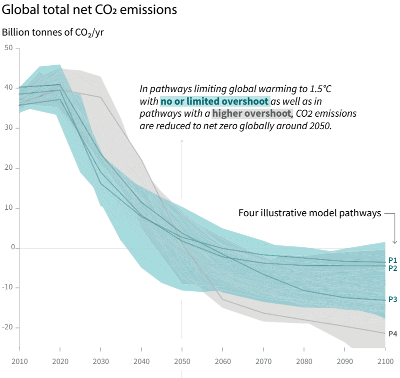


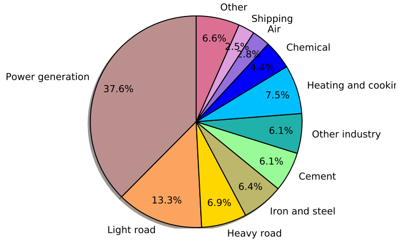


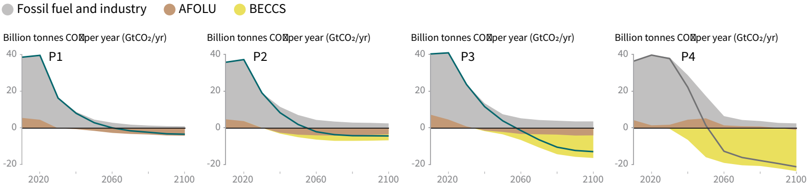


World Energy Outlook 2017, IEA

The energy transition





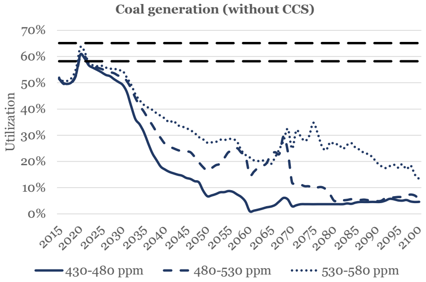


Asset stranding in the energy industry

- To stop climate warming below 1.5 degrees, carbon neutrality must be achieved before 2060.

- Since fossil-fuel power generators have long lifetimes (35-40 years), committed emissions from existing and planned generators are uncompatible with emission pathways towards decarbonized energy sector.

- This creates a risk of asset stranding, i.e., early retirement or underutilisation.



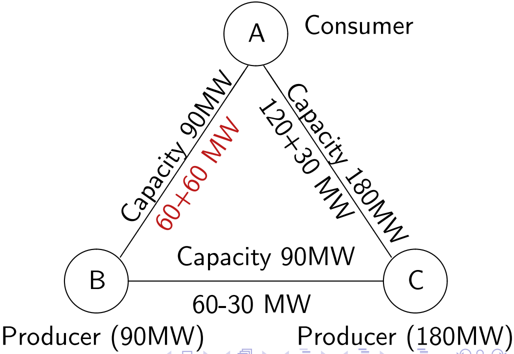
Source: A. Pfeifer et al., "Dead on arrival: Implicit stranded assets in leading IAM scenarios"

Outline

1. The power generation sector
2. Electricity as a commodity
3. Specific risks of intermittent renewable generation
4. Electricity markets and derivative products
5. Modeling and pricing electricity derivatives

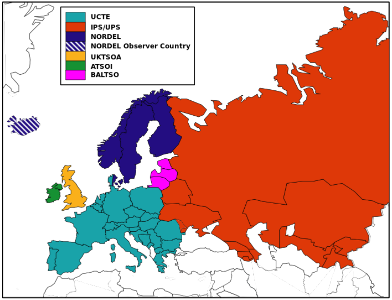
Features of electricity as a commodity

- Electricity (almost) cannot be stored;
- Electricity is a *local* commodity: cross-border capacity is limited, market structure is different in different states
  - Demand must equal supply at all times within each frequency control zone
  - Sum of currents flowing in and out of each node is zero
  - Sum of voltages around each loop is zero
  - Electricity flows through all available paths



Features of electricity as a commodity

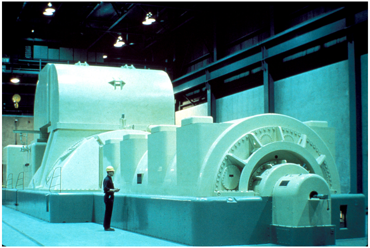
- The electric grid is separated into very large synchronous regions (frequency control zones)
- Complex wholesale market and physical mechanisms are in place to guarantee equilibrium within each zone.



Supply/demand balance and frequency control

- Balancing over very time horizons (seconds) is ensured by frequency control.

- Due to inertia of conventional generators, after a sudden load increase (power plant failure), frequecy of the AC system goes down progressively, allowing the automatic frequency control systems to ramp up production.

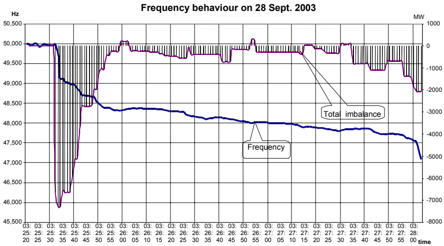


- Renewable generators have no spinning reserve: increased renewable penetration makes frequency control more difficult.

- The effect is stronger in "AC islands" with strong renewable penentration: a study (Connolly et al., 2010) has shown that the energy system of Ireland cannot operate safely with wind energy penetration of over 50% and that the optimal penetration is between 21% and 36%.

Frequncy control

- Frequency is a global charactereistic of power networks
- Frequency stability expresses the balance between generation and load: if load exceeds generation, the speed of turbines starts to drop and the frequency goes down
- In France, admissible frequency range is 50 Hz ± 0.5 Hz. Automatic load shedding starts at 49 Hz.

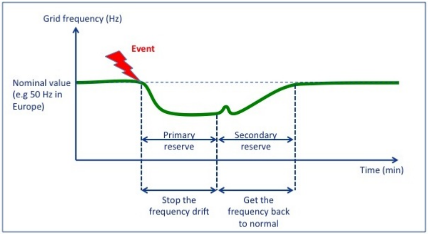


Frequency collapse during the 2003 blackout in Italy.

Source: UCTE report on the 28 September 2003 blackout in Italy

Defense against frequency collapse

- Primary reserve: automatic turbine valve controllers restore supply demand balance at a lower frequency level by increasing output;
- Secondary reserve: capacity of generator units is increased (by the national dispatch center) to bring frequency back to normal.



Source: `cleanhorizon.com`

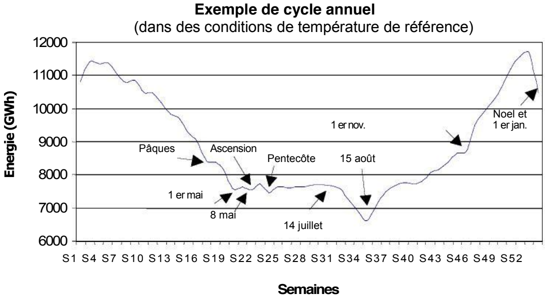
- Tertiary reserve: balancing bids are activated to reconstitute primary / secondary reserve.

In a competitive market, primary/secondary/tertiary reserves must be procured in advance via market mechanisms.

Electricity demand: seasonal trends

Electricity consumption is strongly seasonal with pronounced yearly, weekly and seasonal cycle

- In France, maximal consumption is observed in December and minimal around August 15



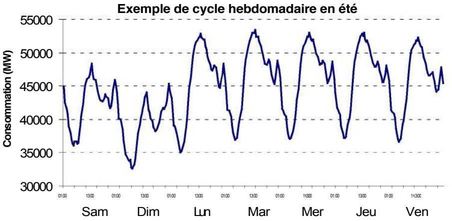
Source: RTE, Consommation française d'électricité, Nov 2014

- The consumption is lower during the week-end and especially on Sunday due to reduced economic activity
- The daily maximum is attained around 1PM in summer and 7PM in winter; the minimal consumption is observed during the night.

Electricity demand: meteorological effects

Electrical consumption is strongly correlated with meteorological conditions.

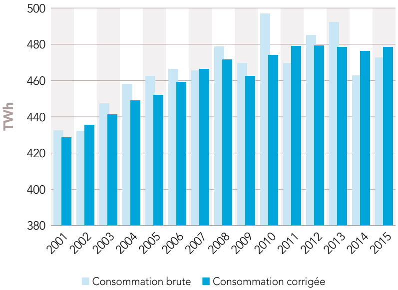
- Temperature: in France, in winter, a variation of $1^{\circ}C$ corresponds to an variation of consumption of 2500 MW. In summer, the variation is about 400 MW due to air conditioning.
- Cloud cover: measured on the scale from 0 to 8. A variation of 1 unit corresponds to a variation of consumption of about 800 MW.



Source: RTE, Consommation française d'électricité, Nov 2014

Electricity demand: non stationarity

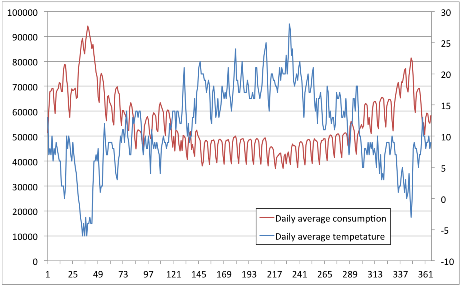
- Electrical consumption is not stationary: average annual consumption has been rising throughout the 20th century, but has stabilized recently in France and has even fallen slightly in the EU over the last 10 years.
- It is expected to rise in future due to electrification of transport and other industries to reduce CO_2 emissions.



Source: RTE, Bilan previsionnel 2016

Electricity demand: consumption spikes

- The main risk for power systems comes from *consumption spikes* during the cold season, which are caused by cold waves.



Data source: RTE web site and www.wunderground.com

(temperature at Paris Orly)

Electricity demand: risks for TSO

- The risks of electricity consumption are related to ensuring supply-demand balance and preventing blackouts.
- In the long term, network structure must be adapted to new consumption patterns, new plants and interconnections must be built.
- In the medium term, sufficient supply margin must be ensured for the cold season, taking into account the possibility of extreme tempetatures.
- In the short term (1-2 days), sufficient production units must be affected to meet demand.
- Following the liberalization of electricity markets, the supply-demand balance is managed through market mechanisms in a (partially) decentralized manner.
- Specific market tools used by TSO to ensure supply-demand balancing are: capacity market, balancing market and system services market (рынок мощности, балансирующий рынок, рынок системных услуг).

Load management

- Individual customers may be encouraged to disconnect loads via specific tariffs (EJP - effacement jour de pointe in France).
- Industrial customers may post load shedding offers on the adjustment markets.
- Utility companies may disconnect certain loads (heating, air conditioning, electric car chargers) directly via smart meters.

Demand response

- Contract between a consumer and a producer (or a retailer)
- The consumer pays a lower fare for power all the days of the year except on a certain days (or periods) decided by the retailer.
- The number of days of price events is determined at inception.
- This form of demand-response contract is named dynamic Time-Of-Use (dToU).

Example: Low Carbon London Pricing trial experiment 2012-2013

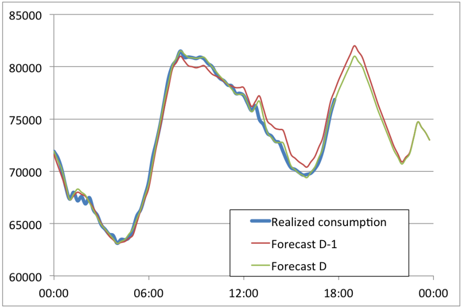
- 5,567 London households with consumption measured at an half hourly time-step on the period from February, 2012 to February, 2014.
- Standard tariff was 14 p/kWh.
- Consumer enrolled in the dToU tariff would pay their power:
  - 11.76 p/kWh on Normal days,
  - 67.2 p/kWh on High price days,
  - 3.99 p/kWh on Low price days.

Demand dispatch

- Demand dispatch refers to direct control of loads by the system operator to optimize network operations.
- The controllable load must be able to shift their consumption in time without affecting the end users. Examples are : water heaters, heating, ventilation, and air conditioning (HVAC), electric vehicle chargers, pool pumps etc.
- Mechanisms for remotely controlling the loads and incentivizing the consumers are presently being developed.

Load forecasting and scenario generation

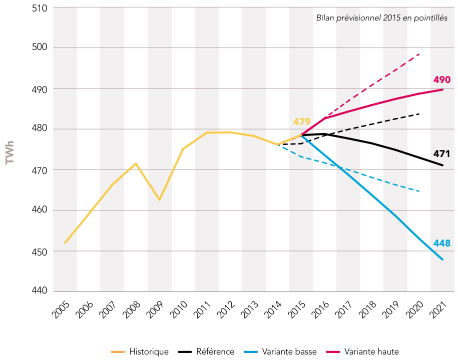
- Short-term forecasting demand (up to 1 week) is based on meteorological forecasts and on historical consumption data.
- Seasonal forecasts are produced with reference meteorological data. Historical simulations may be used for scenario analysis.



Data source: RTE web site

Load forecasting and scenario generation

- Long-term consumption scenarios are produced by RTE based on plausible scenarios of economic activity in different sectors and the evolution of consumption patterns.



Source: Bilan previsionnel RTE 2016

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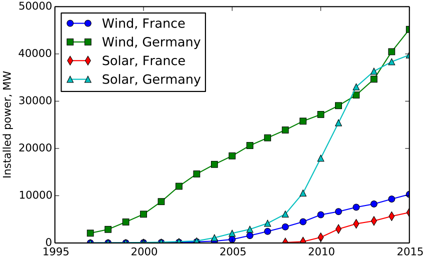
Intermittent renewable production penetration

- European Union targets 20% from renewable energy by 2020 then 27% by 2030 and in some regions renewable penetration is already beyond this target

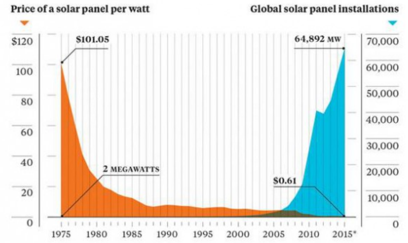


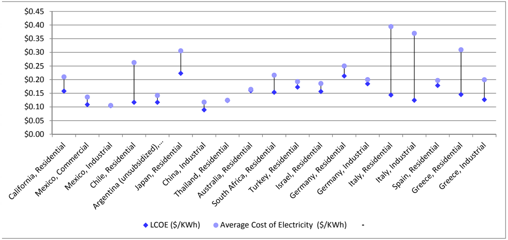
Wind and solar power in France and in Germany. Data source: Wikipedia.

Country	2014 Share	Target
France	18.3%	40% by 2030 27% by 2020
Germany	28.2%	40-45% by 2025 55-60% by 2035 80% by 2050
Denmark	48.5%	50% by 2020 100% by 2050
Italy	33.4%	26% by 2020

Renewable shares and targets in selected EU countries. **Source: REN21 2016 report.**

Solar power economics





Left: cost of solar panels vs. global installed solar power. Source: cleantechnica.com. Right: markets at grid parity, Jan 2014 data. Source: Deutsche bank.

To promote renewable electricity, various support mechanisms are used:

- Feed-in tariffs: used in 19 EU countries

  - in France a wind energy producer sells the generated output to EDF at a fixed price of 8.2 cents per KWh for 15 years (obligation d'achat)

  - The European commission recommends to phase out feed-in tariffs in favor of more market oriented mechanisms

- Feed-in premiums (10 EU countries): the producers sell their output in the market but receive a premium if the market price is below production costs.

  - In France, a 2016 law introduces feed-in premiums for certain renewables (complément de rémunération)

- Quota obligations (green certificates) (6 EU countries): obligation for energy suppliers to purchase a certain percentage of green energy

- Investment support (grants or preferential loans) : 19 EU countries

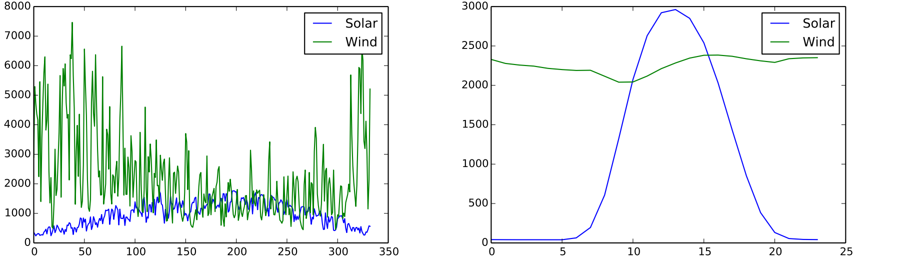
- Tax exemptions – often at the household level: 12 EU countries

Economic framework of renewable energy production

- To encorage the producers to reveal their true costs, support mechanisms are sometimes allocated on a competitive basis (tenders)
- Excessive or poorly implemented support mechanisms drive conventional flexible producers out of the market, reducing spare capacity and system stability
- This has led to the understanding that spare capacity is a commodity and should be financially compensated
- In France spare capacity of producers is certified by RTE and can be bought by suppliers who have the obligation to possess spare capacity matching the peak load of their clients (capacity mechanism)
- An organized capacity market has been launched by EPEX Spot in December 2016

Risks of intermittent renewable production: contingencies

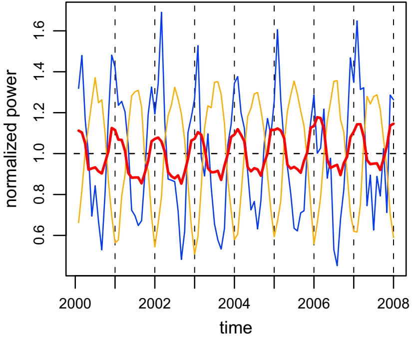
- Production from renewable sources is intermittent and possesses strong daily and annual seasonality creating new constraints on storage capacities in the network to balance supply and demand



Left: average daily wind and solar power production in France in 2016, MW. Right: daily pattern of wind and solar production, 2016 average, MW. Data source: RTE.

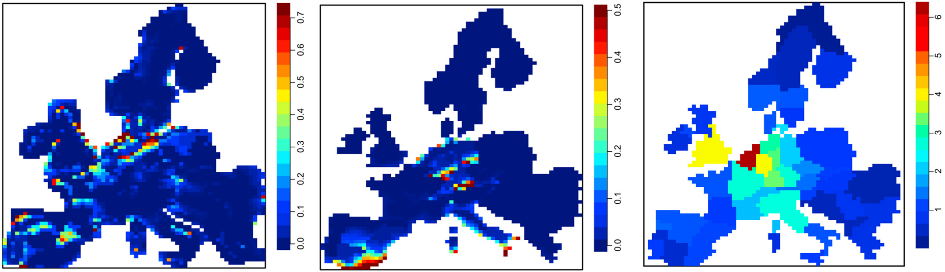
Risks of intermittent renewable production: contingencies

Normalized monthly wind power generation (blue), solar power generation (orange) and load (red) aggregated over Europe. Source: Heide et al. (2010)



Risks of intermittent renewable production: contingencies

- Production from renewable sources is spatially distributed in a non-uniform manner, creating new constraints on transmission capacities in the network to balance supply and demand

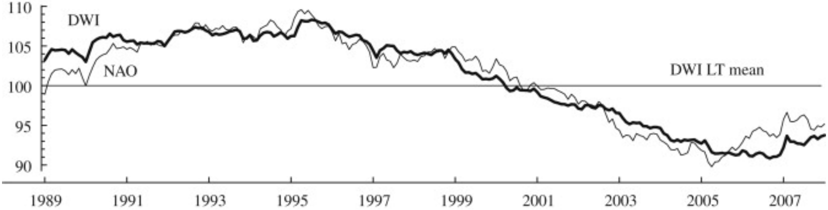


From left to right: expected wind power capacity in 2020, expected solar power capacity in 2020, average annual load.

Source: Heide et al., Seasonal optimal mix of wind and solar power in a future, highly renewable

Risks of intermittent renewable production: contingencies

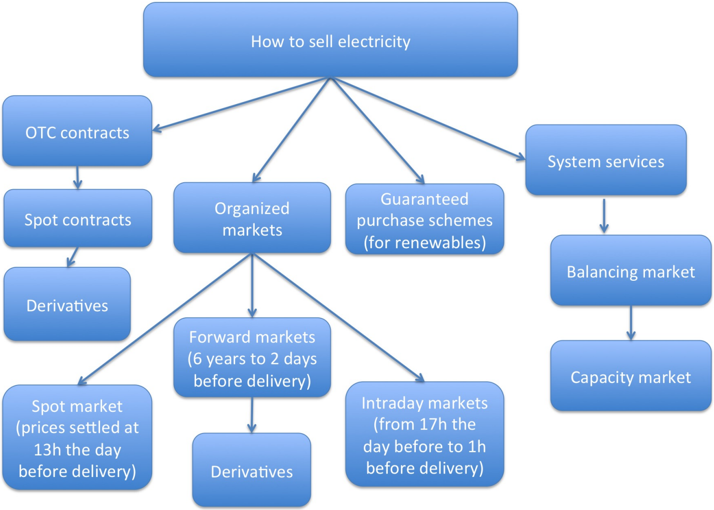
Non-stationarity: due to decadal variations of wind and climate change effects, wind potential may be over-estimated leading to lower than expected profitability of wind farms.



Danish wind index vs. the North Atlantic Oscillation index (difference in pressure between Açores and Iceland). Source: N. Boccard, Capacity factor of wind power: realized values vs. estimates. Energy Policy, 2009

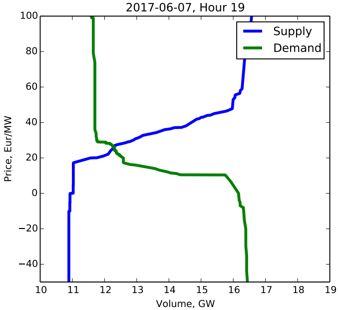
Outline

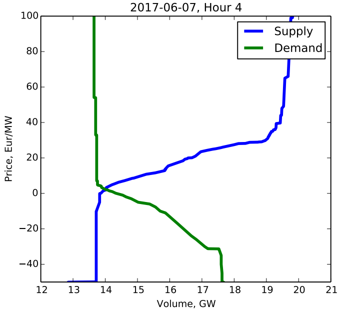
How to sell electricity



The spot (day-ahead) market (рынок на сутки вперед)

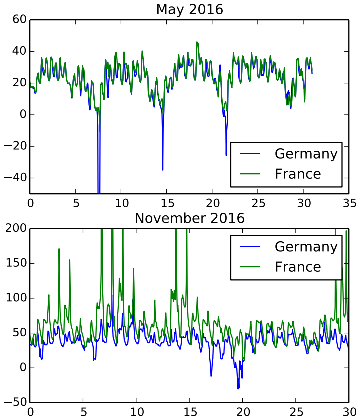
- One of the main trading venues for electricity is the day-ahead market (EPEX Spot in France/Germany).
- In this market trading happens only once: participants submit bids for specific hours of blocks of the next day until 12:00, then at 12:55 the price is fixed and market clears.





Market coupling mechanism

- Each country has its own day-ahead market, but due to market coupling prices in different countries coincide in absence of binding transport constraints
- As long as interconnection capacity permits, demand in one market may be matched by supply in any other market
- If the transport constraints become binding, the prices decouple

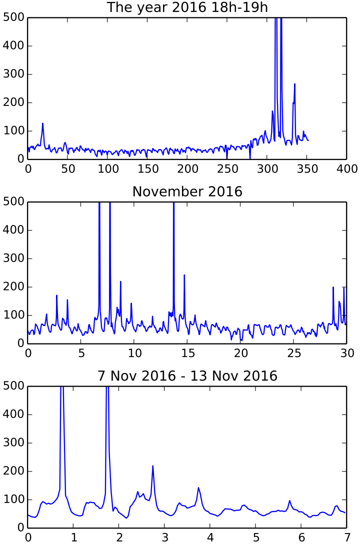


Day-ahead prices in France and Germany. In May, prices are coupled almost all the time, except during negative spikes in Germany. In November, prices are decoupled. Data source: `transparency.entsoe.eu`

Features of spot electricity prices

- Spot electricity prices possess daily, weekly and annual seasonality
- Prices are highly correlated with consumption and in countries where electricity is used for heating / air conditioning, with the temperature
- Due to non-storability, prices exhibit spikes which occur, e.g., in case of plant outage, especially in winter

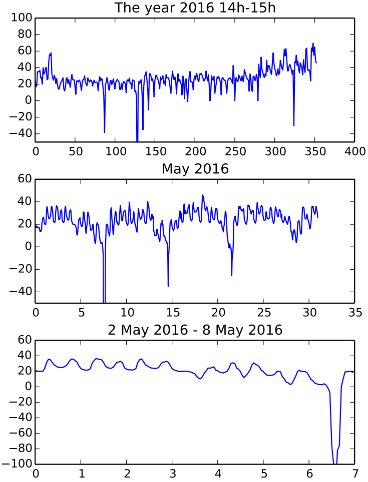
Day-ahead prices in France. Data source: `transparency.entsoe.eu`



Features of spot electricity prices

- Negative prices: since it is costly to shut down coal-fired and nuclear plants, producers are ready to pay to keep the plant running
- This phenomenon is particularly important in Germany due to the large-scale production from renewable sources (at zero marginal cost)

Day-ahead prices in Germany. Data source: `transparency.entsoe.eu`

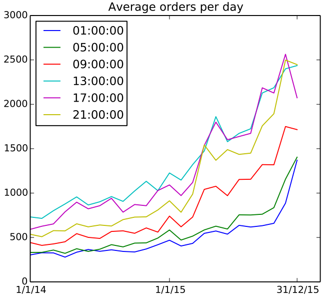


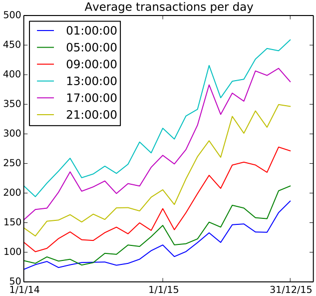
The intraday market

The intraday market opens at 15h and allows continuous trading for each hour/quarter-hour of the next day, up to 30 minutes before delivery.

Every delivery hour of every day corresponds to a different product: the life time of a single product is from 9 to 32 hours.

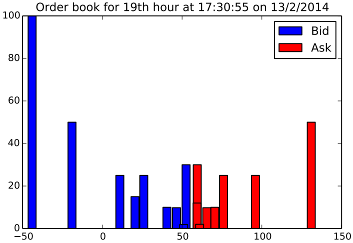
Market liquidity is improving but remains relatively low.





The intraday market

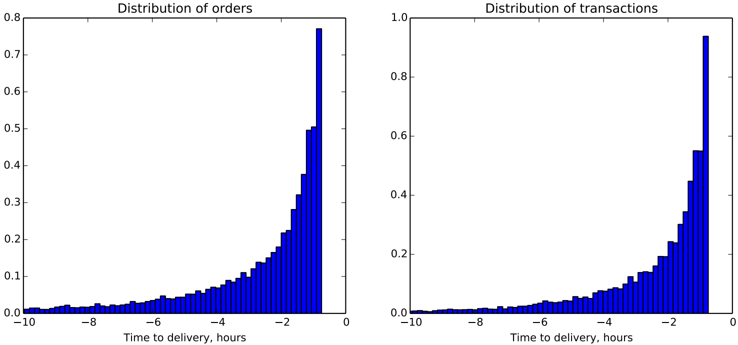
- Trading in intraday markets is order book-based, with a separate order book for each delivery hour.
- Each country has a separate intraday market, but the markets are coupled: if transmission capacity exists, traders in any market see the orders from other markets in their order books.



Intraday electricity markets are gradually acquiring the characteristics of other high-frequency markets with automated trading, optimal execution algorithms, presence of arbitrageurs, price manipulation attempts etc.

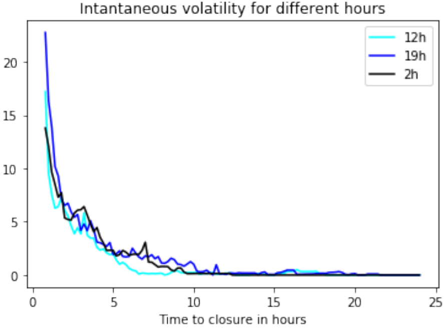
Intraday market liquidity patterns

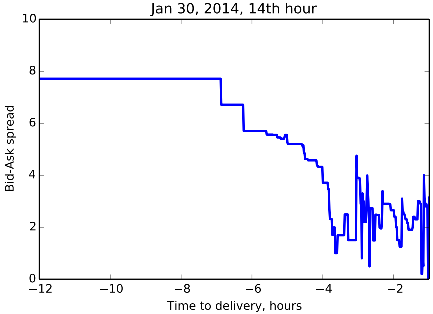
Liquidity only appears a few hours before delivery.



Distribution of orders/transactions as function of time to delivery for all contracts expiring in February 2015.

Bid-ask spread and volatility

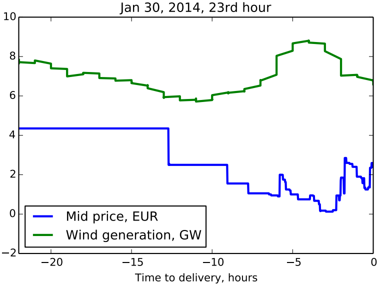


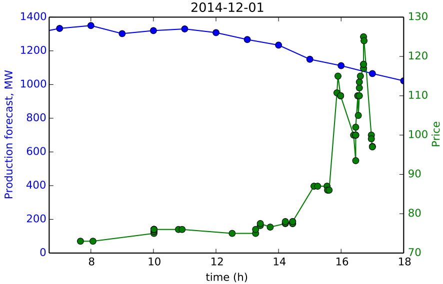


Left: (Normal) volatility averaged over all days of February 2014 (kernel estimator, source: L. Tinsi). Right: bid-ask spread evolution on a typical day.

The intraday market

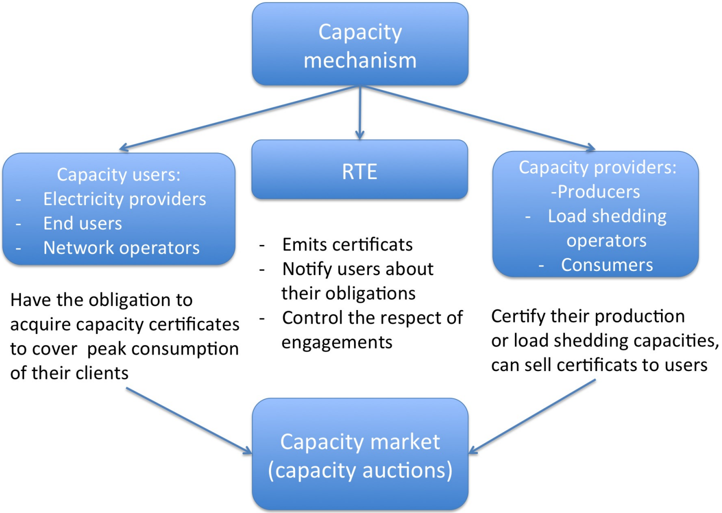
The development of intraday markets has been fueled by the expansion of intermittent renewables: prices are correlated with renewable production forecasts.





Left: dynamics of intraday price vs. wind production in Germany. Right: dynamics of intraday price for the 19th hour of 12 Dec 2014 vs. the forecast of wind power production in France. Data source: EDF and RTE

The capacity mechanism/market (рынок мощности)



The capacity mechanism and capacity market

- Annual certification for the coming year
- Via the 'certification perimieter responsible entity', financially responsible for capacity violations
- Two certification methods:
  - Standard method: based on realized values observed during peak periods
  - Normative method (for renewable energies): based on historical production multiplied by a coefficient
- Below 1MW: compulsory agregation
- Below 100MW: possible agregation

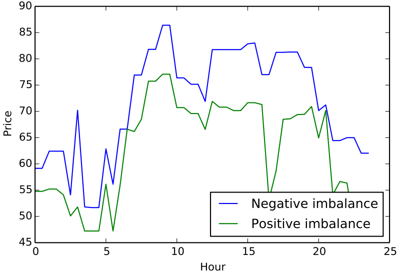
Other	4223 MW
Biomass	77.6 MW
Industrial waste	1.5 MW
Load shedding	1740.8 MW
Onshore wind	2004.3 MW
Hydro/river	4469.7 MW
Gaz/coal	8662.4 MW
Hydro/lake	5584.4 MW
Multi-energy	4659.4 MW
Nuclear	55140.6 MW
Pumped storage	3515.9 MW
Oil	2714.3 MW
Solar	232.8 MW

The Balance Responsible Entity system

- Balance Responsible Entities (BRE, responsables d'équilibre) are basic agents of the French electricity markets
- A BRE, declares to RTE its balance perimeter: portfolio of activities such as
    - Physical sites consuming or generating powe
    - Purchases and/or sales on the power exchanges operating in France;
    - Purchases and/or sales of electricity from/to counterparts;
    - Energy exports and/or imports;
    - Sales of energy to RTE to compensate losses.
- All energy production / consumption must be affected to a balance perimeter of a BRE
- All imbalances within the balance perimeter are compensated to RTE using the imbalance price.

Imbalance prices

- In case of system imbalance, the network operator compensates over-producing agents and applies a penalty to under-producing agents.
- The compensation price and penalty are fixed to enable the network operator to recover the cost of using additional generation.

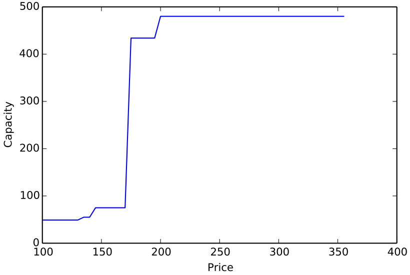


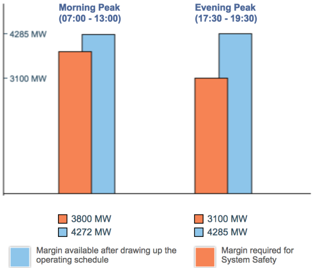
	Balancing trend > 0	**Trend** < 0	**Trend** $= 0$
Imbalance > 0: actor is paid	S	$\min(S, \frac{P}{1+k})$	S
Imbalance < 0: actor pays	$\max(S, P(1+k))$	S	S

Here S is the EPEX Spot price, P is the weighted average price of the balancing and $k = 0.08$.

The balancing mechanism (балансирующий рынок)

- The balancing mechanism (adjustment market) allows the network operator (RTE) to ensure precise overall balance between generation and consumption for the entire system (reconstitute primary and secondary reserve).
- Market players submit bids for increasing production (or reducing consumption).





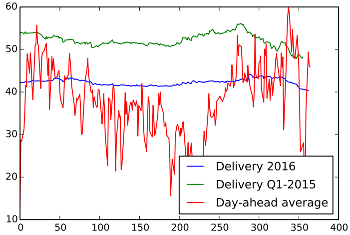
Capacity / price curve (left) and available margins on 15/12/2016. Data: RTE

The forward market

- Electricity futures contract are traded in the European Energy Exchange (EEX).
- Since electricity is a flow commodity, for each contract, a delivery period is specified. For the German market, EEX offers futures for 6 next years, 11 next quarters, 9 next months, 4 next weeks 2 weekends and 8 days.
- Future contracts come in 3 different flavors: base-load (every hour), peak-load (7h-20h Mon-Fri) and off-peak load.
- This allows to maintain reasonable liquidity while enabling market participants to hedge their positions precisely.

The forward market

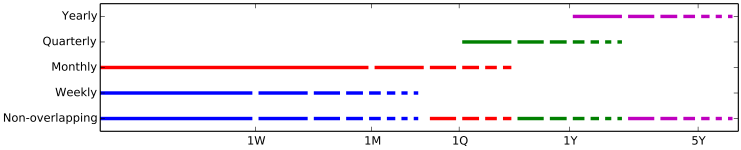
- Futures prices are much less volatile than spot prices, especially for longer delivery periods.
- Due to non-storability of energy, futures prices are not correlated with spot prices and one cannot speak of convergence of futures prices to spot prices.
- Futures for winter delivery are more expensive.



2014 yearly and quarterly base-load futures prices compared to daily average of day-ahead prices. Data source: EDF.

Electricity future contracts

- An electricity future contract (swap) specifies a delivery period
- A future with delivery between T_1 and T_2 settles financially against the average day-ahead price of this period



Electricity derivatives

Standard Calls/Puts on electricity futures are traded in power exchanges such as EEX. They can be valued as standard financial options since the underlying is liquidly traded.

Other options allowing to transfer energy risks are traded over the counter:

- Fuel spreads mimick the profit of a power plant at a given moment in time: the clean fuel spread option pays

$$\left(S^e_T - hS^f_T - gS^c_T\right)^+ ,$$

where

  - S^e is the spot price of electricity;
  - S^f is the spot price of fuel (e.g., gas or coal);
  - S^c is the price of carbon emission allowances;
  - h is the heat rate of the plant;
  - g is the emission rate of the plant.

Electricity derivatives

- Cross-border transmission rights are spread options on the price differential of two neighboring countries (e.g., France vs. Germany). Their pricing is complexified by market coupling (when markets are coupled the spread is zero).
- A tolling agreement mimicks the operation of a power plant over time: it pays

$$\int_0^T \left(S_t^e - hS_t^f - gS_t^c\right)^+ dt.$$

- A swing option is a flexible delivery contract which mimicks a hydroelectric reservoir: the buyer has the right to receive energy (at most $\bar{q}$) on a certain number of days N during a period of time T subject to the constraint that the total consumed power is between $\underline{Q}$ and $\overline{Q}$.

Outline

Commodity futures

- For financial futures, cash and carry arbitrage yields

$$F_t(T) = e^{r(T-t)} S_t.$$

- In presence of storage cost c per unit of time and underlying,

$$F_t(T) = e^{(r+c)(T-t)} S_t,$$

- and for underlyings which cannot be sold short,

$$F_t(T) = e^{(r+c-y)(T-t)} S_t,$$

  where $y \geq 0$ is the "convenience yield" per unit of time and underlying.

Electricity cannot be stored at large scale at reasonable cost, so this relationship breaks down and one needs to model jointly the spot and future prices.

Electricity future contracts

- An electricity future contract (swap) specifies a delivery period
- A future with delivery between T_1 and T_2 settles financially against the average day-ahead price of this period
- Three modeling approaches:
  - Introduce and model a fictitions instantaneous delivery contract

$$F_t(T_1, T_2) = \frac{1}{T_2 - T_1} \int_{T_1}^{T_2} f_t(T) dT;$$

    $\Rightarrow$complicated dynamics for swap prices which are the underlying of options;
  - Model directly the swap prices

    $\Rightarrow$ complicated constraints on the volatility structure;
  - Model the spot price and compute forward prices as risk-neutral expectations of spot price

    $\Rightarrow$ calibration to the initial forward curve difficult

Modeling instantaneous-delivery contracts

Assume a multifactor log-normal dynamics for $f_t(T)$ under a risk-neutral probability measure $\mathbb{Q}$:

$$\frac{df_t(T)}{f_t(T)} = \sum_{i=1}^{n} \sigma_i(t,T)dW_t^i = \sigma^T(t,T)dW_t,$$

where W^i are independent Brownian motions (risk factors) and σ_i are risk factor volatilities.

- Log-normal modeling may be used for long-dated forwards which are not as volatile as the spot
- The number of factors in electricity markets is quite high since forwards of different maturities are loosely coupled
- This modeling is not compatible with lognormal swap price dynamics, often assumed by the market

Implied spot price dynamics

The spot price may be recovered as $S_t = \lim_{T \to t} f_t(T)$. Itô formula yields:

$$\frac{dS_t}{S_t} = \left(\partial \ln f_0(t) - \frac{1}{2}\sigma^2(t,t) + \int_0^t \sigma^T(s,t)\partial_2\sigma(s,t)ds + \int_0^t \partial_2\sigma(s,t)^T dW_s\right) dt$$
$$+\sigma^T(t,t)dW_t.$$

- The spot price is not martingale under $\mathbb{Q}$ because it is not traded;
- The spot price dynamics may not be Markovian

Example

Assume an exponential volatility structure: $\sigma_i(t,T) = \sigma_i e^{-\lambda_i(T-t)}$. Then,

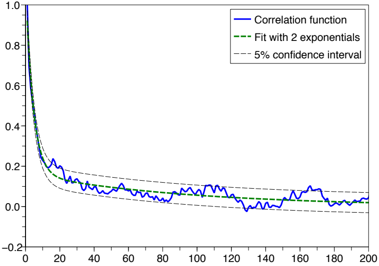
$$S_t = f_0(t)\exp\left(-\frac{1}{2}\int_0^t \|\sigma(s,t)\|^2 ds + \sum_{i=1}^n X_t^i\right)$$

where

$$X_t^i = \sigma_i e^{-\lambda_i t}\int_0^t e^{\lambda_i s} dW_s^i \quad \Rightarrow \quad dX_t^i = -\lambda_i X_t^i + \sigma_i dW_t^i$$

$\Rightarrow$ the spot price is the exponential of a sum of Ornstein-Uhlenbeck processes (may be used to model price spikes)

The graph shows the autocovariance function in the of spot price in Germany fitted with sum of 2 exponentials

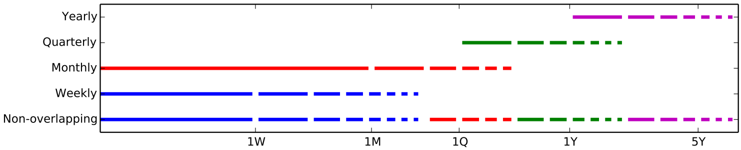


Modeling swap contracts

- An alternative is to model directly the dynamics of traded swap contracts.
- In the presence of overlaps, dynamics is constrained by

$$F_t(T_1, T_2) = \frac{1}{T_2 - T_1} \int_{T_1}^{T_2} f_t(T)dT.$$

- We choose to model the non-overlapping contracts, discarding some information.



A similar log-normal model $dF_t(T_1, T_2) = F_t(T_1, T_2)\Sigma^T(t, T_1, T_2)dW_t$ may be used.

Reduced-form price models

In reduced-form spot price models, one models the day-ahead electricity price directly with a Markov process, and the forward price is deduced by risk-neutral expectation

Usually one models the daily average since intraday structure is complex and irrelevant for forwards

Reduced-form spot price models must respect the following “stylized features”:

- Seasonality;
- Mean reversion;
- Spikes and non-Gaussian behavior.

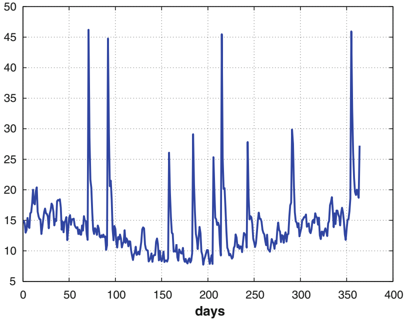
Cartea and Figueroa (2005) model

$$\ln S_t = g(t) + Y_t$$

$$dY_t = -\alpha Y_t dt + \sigma(t) dW_t + J \cdot dq_t$$

where

- $g(t)$ is a deterministic seasonality;
- J is a log-normal proportional jump size;
- q is a Poisson process of jump times.



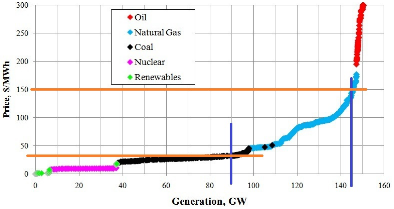
Price trajectory in the Cartea and Figueroa model. Source: Aïd (2015).

In this model, forward prices can be computed explicitly under deterministic market price of risk assumption.

Structural spot price models

- Unlike stock price process which are hardly predictable, electricity prices are related to a multitude of observable factors: consumption, fuel prices, plant outages etc.
- Structural models focus on the price formation mechanism and aim to predict day-ahead prices based on the available information.

- In demand-based models the spot price is obtained by matching a constant supply function with a random inelastic demand.
- In stack-curve models, the supply function is constructed from unit costs and capacities of different generation technologies.



Example of generation stack

Source: **B. Posner** / psu.edu

Demand-based models

- The demand for electricity is described by a stochastic process:

$$D_t = \overline{D_t} + X_t,$$

$$dX_t = (\mu - \lambda X_t)dt + \sigma dW_t,$$

where $\overline{D_t}$ is the seasonal component and X_t is the stationary stochastic part.

- The price is obtained by matching the demand level with a deterministic supply function which must be nonlinear to account for spikes.
- Barlow (2002) proposes

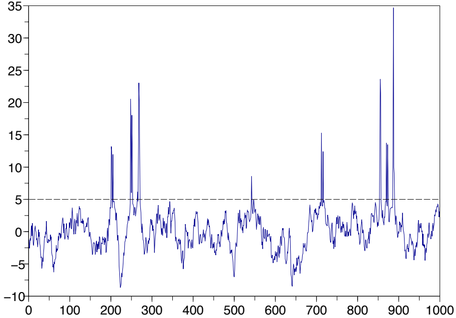
$$P_t = \left(\frac{a_0 - D_t}{b_0}\right)^{1/\alpha}$$

for some $\alpha > 0$.

- Kanamura and Ohashi (2004) suggest a "hockey stick" profile

$$P_t = (a_1 + b_1 D_t)1_{D_t \leq D_0} + (a_1 + b_1 D_t)1_{D_t > D_0}.$$

Demand-based models



Spot price trajectory in the demand-based model by Kanamura and Ohashi (2004).

In these models, spikes can only be caused by surges in demand, while in electricity markets spikes can also be due to sudden changes in supply, such as plant outages.

Stack curve model of Aïd (2009)

- The electricity demand D_t can be satisfied with n different technologies;
- Each technology has available capacity C_t^i and fuel cost $h_i S_t^i$, where S^i is the fuel price and h_i is the heat rate;
- The marginal fuel cost is

$$\widehat{P}_t = \sum_{i=1}^n h_i S_t^i \mathbf{1}_{D_t \in I_t^i}, \qquad I_t^i = \left(\sum_{k=1}^{i-1} C_t^k, \sum_{k=1}^{i} C_t^k \right]$$

- The spot price depends on the marginal fuel cost and the reserve margin:

$$P_t = g(R_t) \times \widehat{P}_t, \qquad R_t = \sum_{i=1}^n C_t^i - D_t$$

where g is the *scarcity function*:

$$g(x) = \min\left(\frac{\gamma}{x^\nu}, M\right) \mathbf{1}_{x>0} + M \mathbf{1}_{x \le 0}.$$

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Call and put options

- In the EEX market, call and put options on 6 monthly swaps, 6 quarterly swaps and 3 yearly swaps (base-load) are traded
- For every swap, a single maturity, shortly before the swap expiry is offered, with a variety of strikes
- The underlying of each option is therefore traded throughout the lifetime of the option
- Risk-neutral pricing approaches are therefore justified

Consider the pricing of a call option on an electricity swap contract with pay-off

$$(F_T(T_1, T_2) - K)^+.$$

Its price at t is given by

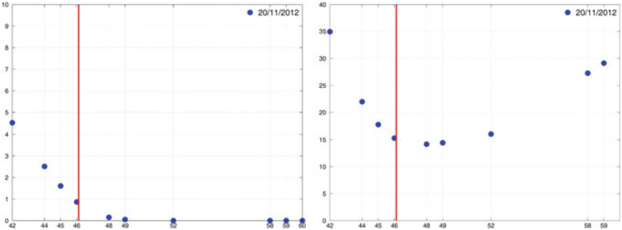
$$V_t = e^{-r(T-t)}\mathbb{E}^{\mathbb{Q}}[(F_T(T_1, T_2) - K)^+|\mathcal{F}_t]$$

Call and put options

- In a log-normal model for the forward $f_t(T)$: pricing by Monte-Carlo.
- In a log-normal model for the swap price $F_t(T_1, T_2)$: Black's formula

$$V_t = e^{-r(T-t)} F_t(T_1, T_2) N(d_1) - e^{-r(T-t)} K N(d_2),$$

$$d_{1,2} = \frac{\log \frac{F_t(T_1,T_2)}{K} \pm \frac{1}{2} v_{t,T}}{\sqrt{v_{t,T}}}, \quad v_{t,T} = \int_t^T \|\Sigma(s, T_1, T_2)\|^2 ds.$$



Prices and implied volatilities of year-ahead base-load futures options with expiry on Jan 2013, as of Nov 20, 2012. Source: R. Aïd (2015).

Spread options on forward contracts

- Most commodity spread options are written on forward contracts
- The underlyings are liquidly traded and risk-neutral valuation may be used

Consider two forward contracts with risk-neutral dynamics

$$dF_t^1 = F_t^1 \sigma_1(t) dW_t, \qquad dF_t^1 = F_t^2 \sigma_2(t)(\rho dW_t + \sqrt{1-\rho^2} dW_t'),$$

where σ_1 and σ_2 are deterministic volatility functions and W and W' are independent standard BMs

Magrabe's formula gives the price of the zero-strike option on $F_T^1 - F_T^2$:

$$e^{-r(T-t)} \mathbb{E}^{\mathbb{Q}}[(F_T^1 - F_T^2)^+ | \mathcal{F}_t] = e^{-r(T-t)}(F_t^1 N(d_1) - F_t^2 N(d_2)),$$

$$d_{12} = \frac{\log \frac{F_t^1}{F_t^2} \pm \frac{1}{2} v_{t,T}}{\sqrt{v_{t,T}}}, \quad v_{t,T} = \int_t^T (\sigma_1^2(t) + \sigma_2^2(t) - 2\rho\sigma_1(t)\sigma_2(t)) ds$$

Kirk's formula for non-zero strike spreads

When $K \neq 0$, Magrabe's formula no longer holds and no explicit expression for spread option price is available

Kirk's formula is an empirical approximation obtained by replacing the log-normal dynamics of F^2 with a shifted log-normal one:

$$d(\widehat{F}_t^2 + K) = (\widehat{F}_T^2 + K)\hat{\sigma}_2(\rho dW_t + \sqrt{1-\rho^2}dW_t'), \quad \hat{\sigma}_2 = \sigma_2 \frac{F_0^2}{F_0^2 + K}$$

Applying Magrabe's formula leads to the approximation

$$e^{-rT}\mathbb{E}^{\mathbb{Q}}[(F_T^1 - F_T^2 - K)^+] \approx e^{-rT}(F_0^1 N(d_1) - (F_0^2 + K)N(d_2)),$$

$$d_{12} = \frac{\log \frac{F_0^1}{F_0^2+K} \pm \frac{1}{2}\sigma_K^2 T}{\sigma_K\sqrt{T}}, \quad \sigma_K^2 = \sigma_1^2 + \hat{\sigma}_2^2 - 2\rho\sigma_1\hat{\sigma}_2.$$

Example: cross-border transmission rights

- Cross-border transmission rights can be purchased at an auction for a period of one year, one month or one day
- The allocated capacities can then be nominated within the limits set by the programming authorisations for each hour of the day, before 15:30 of the previous day (day-ahead prices already known).
- Pay-off of a cross-border transmission right with unit capacity from region with price S^2 to region with price S^1 for a total of T hours:

$$\sum_{t=1}^{T} n_t (S_t^1 - S_t^2)^+,$$

  with n_t is the programming authorisation in percentage of the capacity.
- Difficulty: n_t not known in advance; the underlyings are day-ahead prices which are not traded assets
- Corresponding futures contracts are not traded either