

<b>List of talks: Bachelier Colloquium, Metabief, January 13-18 2020</b>
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**Belmesnaoui Aqzzouz, University Mohammed V of Rabat, Morocco.**

On general equilibrium in a financial market model whose commodity space is the portfolio space.

We consider a model security market with finite investors and infinitely many securities. In this situation it is possible that optimal portfolio allocations and equilibrium portfolio allocations do not exist. We use the theory of General equilibrium of Arrow-Debreu in infinite dimension to prove that such allocations exist if the Portfolio space is a vector lattice.

**Cagin Ararat, Bilkent University, Turkey.**

Set-valued backward stochastic differential equations.

Motivated by the connection between univariate dynamic risk measures and backward stochastic differential equations, we start building a theory for set-valued backward stochastic differential equations (SV-BSDE). As a first attempt in this direction, we propose a simple SV-BSDE where the values of the driver function are compact sets. Our main result establishes the well-posedness of the SV-BSDE under certain conditions on the driver. The key ingredients of the proof are a recent set-valued martingale representation theorem by Kisielewicz as well as two useful estimates that we prove for a priori solutions of the SV-BSDE. Joint work with Jin Ma and Wenqian Wu.

**Dirk Becherer, Humboldt University Berlin, Germany.**

Optimal execution with a view on price trends for transient multiplicative impact.

We solve the optimal trade execution problem to turn over a large financial asset position in finite time in an illiquid market where mechanical price impact is transient and possibly non-linear (in log-prices), like in models by Bouchaud, Lillo, Gatheral and others. The impact model can be seen as a multiplicative variant of the models by Obizhaeva-Wang (2013) or Predoiu-Shaikhnet-Shreve (2011), but for impact being in relative terms or percentage (Bertsimas-Lo (1998)) and with non-negative asset prices. Our analysis admits for the drift of the unaffected asset price to be non-zero, i.e. a basic directional view about the price trend (Almgren-Chriss (2000)). We derive a rather explicit description in terms of characteristic curves for the (regular) three-dimensional free boundary surface that determines the no-action regions of the singular control problem. While the boundary description is almost as simple as in Obizhaeva-Wang, our

solution requires a very different analysis as our problem lacks an apparent convexity structure. Maybe surprisingly, a key step to show global optimality has been to show at first a local optimality of a candidate free boundary by variational arguments, similarly to B.Bilarev-Frentrup (2018, F.S.), in a suitably re-parameterized state space. For the application problem, the results permit to investigate phenomena, like for instance: a) optimal (non-trivial) round-trips to exploit views on price trends (drift), b) front- or back-loading of optimal (sell-) trade execution strategies when price trend is upwards or downwards, respectively, or c) the qualitatively different nature of optimal trading strategies and non-monotone dependence of their profitability on the resilience (transience) of market impact.

**Yana Belopolskaya, SPbGASU, PDMI RAS, St. Petersburg, Russia.**

Probabilistic interpretation of parabolic conservation laws.

We consider the Cauchy problem for systems of parabolic equations which arise as mathematic models for conservation and balance laws in physics, chemistry, biology and so on. Among those systems there are diffusion-reaction systems, cross-diffusion systems and many others. We divide these systems in two classes - the first class includes systems with diagonal entrance of higher order terms and the second includes systems with nondiagonal entrance of these terms. We treat both classes as systems of forward Kolmogorov equations and construct stochastic systems associated with them. In most cases systems from the first class are systems w.r.t functions, while systems from the second class should be treated as systems in measures (in regular cases) or in densities of these measures (in singular cases).

We show that the first class of systems admits a probabilistic representation of the Cauchy problem solution in terms of one underlying diffusion process and its multiplicative operator functionals. This gives a possibility to construct a system of SDEs (one nonlinear and one linear) with coefficients depending on the required function and to derive an integral equation for this function as a closing relation for the stochastic system. Among other results we can construct a probabilistic justification of the vanishing viscosity method that allows to obtain a solution of a system of nonlinear hyperbolic equations.

To construct a probabilistic counterpart of a system from the second class we develop an alternative technique showing that in this case we have to deal with a number of diffusion processes satisfying SDEs with coefficients depending on the required functions and derive the closing relation from some generalizations of the Feynmann-Kac formula. We present some examples of probabilistic interpretations of systems from both classes.

The support of the RSF Grant 17-11-01136 is gratefully acknowledged

**Elena Boguslavskaya, Brunel University, London, England.**

Portfolio optimisation with multiple mean-reversion.

How should one construct a portfolio from multiple mean-reverting assets? Should one add an asset to the portfolio even if it has zero mean reversion? We consider a position management problem for an agent trading multiple mean-reverting assets. We solve an optimal control problem for an agent with power utility, and present a semi-explicit solution. The nearly explicit nature of the solution allows us to study the effects of parameter sensitivity. This is a joint work with Dmitry Muravey and Michael Boguslavsky.

**Svetlana Borovkova, Vrije Universiteit Amsterdam, Netherlands.**

Special session: Learning with sentiment: machine Learning with news and social media sentiment for equity market investment and trading.

Successful forecasting of financial markets has long been the “holy grail” of researchers and practitioners, as this can lead to profitable trading strategies. With advances in machine learning, the quest for such forecasting and trading algorithms has received new attention. However, for machine learning algorithms to be successful, they need to incorporate external real time information that drives prices of financial assets. Two such sources of information are news and social media content related to publicly traded companies, commodities or indices. In this talk, I will discuss how stock prices respond to sentiment in news and show that this response is quite universal, regardless of the size, liquidity of a stock or trading period. I will also describe several examples of recent use cases where sentiment in news and social media has been successfully used in machine learning models (such as neural networks and random forests) to enhance their forecasting performance. These examples range from intraday trading of major market indices to daily trading of individual stocks.

**Evgeny Burnaev, Skolkovo Institute of Science and Technology, Moscow, Russia.**

Special session: Financial time series analysis of SV model by deep Monte Carlo.

We apply the Monte Carlo algorithm to the financial time series analysis of the stochastic volatility (SV) model. The algorithm is based on deep neural autoregressive flow. We compute parameters of the SV model from the financial data and compare the results from the algorithm with those from the Metropolis algorithm. We find that the algorithm decorrelates the volatility variables faster than the Metropolis algorithm.

**Zeyu Cao, Stony Brook University University, USA.**

SABR type stochastic volatility operator in Hilbert space.

In this paper, we studied the Hilbert space-valued statistical dynamical model of the instantaneous forward interest rate curves with  $D(x) \frac{\partial^2}{\partial x^2}$  as a regularity term, which is first considered and studied by R. Douady, M. Musiela, R. Cont and so forth. In particular, we raised the question: how to construct infinite dimensional version of stochastic volatility which is a Hilbert-Schmidt operator valued stochastic process. In order to handle the SABR-type volatility operators, we propose several kinds of models, the first one focus on the volatility of volatility operators and can be regarded as a direct generalization of the classic SABR model, the second one focus on modeling the correlations among the Brownian motions while the third one is more general, involving more non-linearity and flexibility and is a combination of the first two models. All of them are dependent on the absolute value of the instantaneous forward interest rate curve to the power of  $\beta$ ,  $0 < \beta < 1/2$ . We showed the existence of mild solutions to our models. Our proof is based on Leray-Schauder fixed point theorem and some priori inequalities on the stochastic operator processes we construct. Further, we relaxed the assumptions and studied the regularity of the solutions to some specific models.

**Laurence Carassus, PLV research center, Paris, France.**

Risk-neutral pricing for arbitrage pricing theory.

We consider two infinite dimensional optimization problems motivated by the financial model called Arbitrage Pricing Theory. Using probabilistic and functional analytic tools, we provide first a dual characterization of the super-replication cost. We also show the existence of optimal strategies for investors maximizing their expected utility and the convergence of their reservation prices to the super-replication cost as their respective risk-aversion tends to infinity.

**Tahir Choulli, University of Alberta, Canada.**

NFLVR and optimal consumption under arbitrary random time.

I consider an initial market model  $(S, F, P)$ , and an arbitrary random time  $T$ . Here  $S$  is the discounted assets prices,  $F$  is the public flow of information,  $P$  is a probability measure, and  $T$  might represent a default time of a firm or the death time of agent. In this setting, I will address three main topics. The first topic consists of investigating the NFLVR (No Free Lunch with Vanishing Risk) for the new model resulting from stopping  $S$  with  $T$ , that we denote by the triplet  $(S^T, G, P)$ , where  $G$  is the new flow of information that contains  $F$  and makes  $T$  a stopping time. Here we suppose that the survival probability never vanishes and prove that NFLVR always holds for the stopped model as soon as it holds for the original model. Furthermore, we give a large family of martingale sigma-densities for the stopped model in terms of the sigma-densities of the initial mode. The second main topic consists of Merton's problem up to the random horizon  $T$ , and without bequest (the optimal consumption problem only). For

this problem, we assume that the initial model is complete (in order to single out more efficiently the effect of  $T$ ) and the power utility. For this economic framework, we prove that the optimal consumption depends on the “correlated-risk” between  $T$  and  $S$  only, and it is not affected by the “pure default risks” at all. The classification of risks of the model  $(G, P)$  was achieved in Choulli et al (2015), where we refer the reader for more details about the matter. The third topic lies in the optimal investment problem (basically the Merton’s problem without consumption before maturity) up to the random horizon  $T$ . Again, herein, we suppose that the initial model is complete and we consider the power utility. We prove that, in contrast to the optimal consumption, the optimal portfolio depends on two types of risks generated by  $T$ , namely the “correlated-risk” and the “pure default risk” of type one. These results could not be derived, nor even guessed, without the important result of explicitly and fully describing all deflators for the stopped model  $(S^T, G, P)$  in terms of those of the initial model  $(S, F, P)$ . In my talk, I will start by this latter result. This talk is based on works with my PhDs (former and current), Sina Yansori, Ferdoos Alharbi and Safa’ Alsheyab.

**Stephane Crepey, Evry University, France.**

Special Session: Deep XVA analysis.

Since the 2008-09 financial crisis, derivative dealers charge to their clients various add-ons, dubbed XVAs, meant to account for counterparty risk and its capital and funding implications for banks. Two competing XVA paradigms are a replication framework and a cost-of-capital, incomplete market approach. Burgard and Kjaer once dismissed an earlier incarnation of the Albanese and Crépey holistic, incomplete market XVA model as being elegant but difficult to solve explicitly. In this talk we show that this model, set on a forward/backward SDE formulation, is not only well-grounded economically, but also able to be solved efficiently using GPU computing combined with machine learning methods in a whole bank balance sheet context. We calculate the mark-to-market cube using GPU computing and the XVA processes using deep learning regression and quantile regression (for the embedded conditional value-at-risk and expected shortfall calculations) schemes.

Joint work with Rodney Hoskinson, ANZ Bank, and Bouazza Saadeddine, Université Paris-Saclay, Evry.

**Christoph Czichowsky, LSE, London, England.**

Rough volatility and portfolio optimisation under transaction costs.

Rough volatility models have become quite popular recently, as they capture both the fractional scaling of the time series of the historic volatility (Gatheral et al. 2018) and the behavior of the implied volatility surface (Fukasawa 2011, Bayer et al. 2016) remarkably well. In contrast to classical stochastic volatility

models, the volatility process is neither a Markov process nor a semimartingale. Therefore, these models fall outside the scope of standard stochastic analysis and provide new mathematical challenges. In this talk, we investigate the impact of rough volatility processes on portfolio optimisation under transaction costs.

The talk is based on joint work with Johannes Muhle-Karbe (Imperial College London) and Denis Schelling (London School of Economics and Political Science).

**Moussa Dabo, Université Paris 1 - Panthéon Sorbonne, France.**

The CMMV pricing model in practice.

Mainstream financial econometrics methods are based on models well tuned to replicate price dynamics, but with little to no economic justification. In particular, the randomness in these models is assumed to result from a combination of exogenous factors. In this paper, we present a model originating from game theory, whose corresponding price dynamics are a direct consequence of the information asymmetry between private and institutional investors. This model, namely the CMMV pricing model, is therefore rooted in market microstructure. The pricing methods derived from it also appear to fit very well historical price data. Indeed, as evidenced in the last section of the paper, the CMMV model does a very good job predicting option prices from readily available data. It also enables to recover the dynamic of the volatility surface.

**Albina Danilova, London School of Economics, England.**

Risk aversion of market makers and asymmetric information.

In this talk I will explore differences and similarities of informed trading in the markets with different types of liquidity providers. In particular, I will present two types of markets: the one populated by perfectly competitive liquidity providers, who take the asset supply as given, and the one populated by market makers, who compete for total demand by posting price schedules. A particular focus will be on the impact of risk aversion of market makers/liquidity providers on the resulting equilibrium price process.

I will demonstrate that, although the resulting equilibrium price is different, the markets are quite similar qualitatively: the demand is mean reverting and the price process exhibits price reversal in both markets. Furthermore, in both markets an increase in risk aversion leads to lower market depth, less efficient prices, stronger price reversal and slower convergence to fundamental value. The endogenous value of private information, however, is non-monotonic in risk aversion and differs in two markets.

**Purba Das, University of Oxford, England.**

Quadratic Variation and Quadratic Roughness.

We study the concept of quadratic variation of a continuous path along a sequence of partitions and its dependence with respect to the choice of the partition sequence. We define the quadratic roughness of a path along a partition sequence and show that, for Holder-continuous paths satisfying this roughness condition, the quadratic variation along balanced partitions is invariant with respect to the choice of the partition sequence. Paths of Brownian motion are shown to satisfy this quadratic roughness property almost-surely. Using these results we derive a formulation of Follmer's pathwise integration along paths with finite quadratic variation which is invariant with respect to the partition sequence.

**Davide De Santis, LSE, London, England.**

Zero-sum stochastic differential games with impulse controls: a stochastic Perron's method approach.

The main object of this work is to apply the stochastic Perron method to zerosum stochastic differential games with impulse controls. We consider a symmetric game in which the two agents are playing feedback impulse control strategies instead of defining them in an Elliot-Kalton fashion. The upper value and lower value functions are naturally associated to two double obstacle partial differential equation (PDE) and, in particular, it turns out that the upper value function's PDE, the upper Isaacs, coincide with the case in which P1, the maximiser, has precedence of intervention over P2, the minimiser, whereas the lower value function's PDE, the lower Isaacs, coincide with the case in which P2 has precedence. Once the upper and lower value functions are defined we suitably characterise the stochastic sub and super solutions of the upper and lower Isaacs as those functions dominated by and dominating the respective Isaacs equation. Then, we show that the infimum of stochastic supersolutions of the upper Isaacs is its viscosity subsolution whereas the supremum of stochastic subsolutions of the upper Isaacs is its viscosity supersolution so that a viscosity comparison result will give us the unique and continuous viscosity solution characterising the value of the game.

**Thomas Deschatre, EDF Lab, France.**

Special session: Deep combinatorial optimization for stochastic control in finance - Application to american options and fixed transaction costs.

Neural networks for stochastic control and associated PDE shows good results especially for problems in high dimensions. Regarding trading and hedging neural networks based algorithms offer an alternative or a complement to traditional stochastic control tools. We use reinforcement learning with policy gradient to solve stopping time problems in finance. A neural network is used to model the probability to have an action (for instance exercise an option or take a position for hedging) as a function of the observed states (prices). The

gradient of the objective function is computed using likelihood ratio method. The method allows to solve american option pricing (even with high dimensions) and hedging with fixed transaction costs. One of the benefits of the method is that it is robust to the dimension of the problem (up to 5 source of randomness tested). One of the most issue is that local minima can be achieved instead of global (which is a well known issue in reinforcement learning).

**Ernst Eberlein, University of Freiburg, Germany.**

A Multiple curve Lévy swap market model.

The global financial crisis which started in 2007 changed the fixed income markets in a fundamental way. Due to a new perception of risk, a number of interest rates, which until then had been roughly equivalent, drifted apart. The basic rates, which are relevant for the interbank market, became tenor-dependent. In the new multiple curve reality classical modeling approaches which are based on arbitrage considerations assuming tenor-independence cannot reflect the market behaviour any more. As a consequence a large variety of modelling approaches of increasing mathematical sophistication has been studied in recent years.

In this paper we develop an arbitrage free multiple curve model for the interest rate market through the specification of forward swap rates. We start with two sets of assets as fundamentals: OIS zero-coupon bonds for all maturities as well as forward rate agreements for all tenors, settlement dates and strikes. In contrast to most of the approaches considered so far where short rates or instantaneous forward rates as well as IBOR rates or forward processes are chosen as the quantities which are to be modeled, we consider here the dynamics of the swap rates directly. This is a very natural approach since on one side we build on OIS bonds and on the other side the mid and long maturity part of the term structure is bootstrapped from swap quotes. The rates are constructed via a backward induction along the tenor structure on the basis of the forward swap measures. Time-inhomogeneous Lévy processes are used as drivers. As an application we derive an explicit Fourier based valuation formula for swaptions. Joint work with Christoph Gerhart and Eva Lütkebohmert.

**Sergei Egorov, Rouen University, France.**

Optimal investment and consumption problem for Lévy markets with transaction costs.

We consider optimal consumption and investment problems for financial markets driven by Lévy processes for the power utility functions using the dynamic programming method. To this end we study the Hamilton–Jacobi–Bellman (HJB) equation by separation variables methods and finally find the classical HJB solution in explicit form. Then we show the corresponding verification theorem for this problem and as a consequence construct the optimal strategies.

Then using the Leland approach, we modify the obtained optimal strategies to take into account transaction costs. Next, we investigate the asymptotic behavior, when the number of revisions tends to infinity. For the modified optimal strategy, we have shown that, the objective function tends to its maximum value, i.e. we have proved that the constructed strategy is asymptotically optimal.

**Meriam El Mansour, Paris Dauphine University and Faculty of Sciences of Tunis, Tunisia.**

Conditional interior and conditional closure of a random set.

We introduce two new concepts of conditional random set in a Banach space: The conditional interior and the conditional closure. The conditional interior is an open version of the conditional core, as recently introduced by Lépinette and Molchanov, and may be seen as a measurable version of the topological interior. The conditional closure generalizes the concept of conditional support. These concepts are useful for applications in mathematical finance and conditional optimization. Joint work with E. Lépinette.

**Eugene A. Feinberg, Stony Brook University, USA.**

Special session: Fatou's lemmas for varying probabilities and their applications to sequential decision making.

The classic Fatou lemma states that the lower limit of expectations is greater or equal than the expectation of the lower limit for a sequence of nonnegative random variables. This talk describes several generalizations of this fact including generalizations to converging sequences of probability measures. The three types of convergence of probability measures are considered in this talk: weak convergence, setwise convergence, and convergence in total variation. We shall also discuss applications of these results to sequential optimization problems with completely and partially observable state spaces.

This talk is based on joint papers with Pavlo Kasyanov, Yan Liang.

**Miryana Grigorova, University of Leeds, England.**

Non linear incomplete market with default: the case of American options.

We present an incomplete market model with default which consists of one risky asset with dynamics driven by two "sources of risk", namely a Brownian motion and a compensated default martingale. Additionally to this feature, the wealth process follows non-linear dynamics with a non-linear driver  $f$ , which allows to incorporate a number of imperfections in the market. We thus face a non-linear incomplete market with default.

In last year's talk, we focused on the superhedging price problem for European options in this framework. In this year's talk, we will address the more involved case of American options.

We will present the case of American options with càdlàg pay-off process, and also the more general case of American options with completely irregular pay-off (not necessarily càdlàg). We will give a dual representation of the seller's (superhedging) price for the American option in terms of the value of a non-linear mixed control/stopping problem. The dual representation involves a suitable set of equivalent probability measures, which we call  $f$ -martingale probability measures. We will also provide two infinitesimal characterizations of the seller's price process: in terms of the minimal supersolution of a constrained reflected BSDE with default and in terms of the minimal supersolution of an optional reflected BSDE with default. If time permits, we will also present results from the buyer's point of view. The talk is based on joint works with Marie-Claire Quenez and Agnès Sulem.

**Nikolay Gudkov, ETH Zurich, Switzerland.**

Application of power series approximation techniques to the pricing and hedging of European style options.

Fourier transforms provide versatile techniques for the pricing of financial derivative securities. In applying such techniques, a typical derivative valuation expression is often written as an inner product of the Fourier transform of the payoff and the characteristic function of the underlying asset dynamics. Some modelling specifications imply that it might be challenging to find a closed-form expression for the characteristic function. In such situations, numerical approximations have to be employed. This paper utilises the power series approximation technique in finding explicit expressions of the characteristic function for the underlying stochastic variables. We analyse the convergence and accuracy of the method in the context of valuing European style options written on underlying securities whose dynamics evolve under the influence of multiple Heston-type stochastic volatilities Heston (1993) and Cox-Ingersoll-Ross stochastic interest rates Cox (1985). The paper contributes to the existing literature four-folds by (i) adapting the valuation technique to long-dated instruments; (ii) providing an adjustment to the series of the points around which the power series expansion is performed; (iii) analysing the performance of different strategies for hedging European call options; (iv) applying the power series approach to the valuation of guaranteed minimum accumulation benefit riders embedded in variable annuity contracts. Our results demonstrate the high computational efficiency of the series approximation method for the computation of derivative prices and hedge ratios.

**Alexander Gushchin, Steklov Mathematical institute, Russia.**

Single jump filtrations and local martingales.

We define a single jump filtration on some probability space corresponding to a random time  $\gamma$  in such a way that  $\gamma$  is a stopping time with respect to this

filtration, every adapted process is deterministic before  $\gamma$ , and the sigma-algebra of events that occur before or at time  $\gamma$  coincides with the sigma-algebra of all measurable sets. We prove a simple characterization of all local martingales in this model. This result seems to be new even in a special case that has been studied in the literature, namely, where the sigma-algebra of all measurable sets is the smallest sigma-algebra with respect to which  $\gamma$  is measurable (and then the filtration is the smallest one with respect to which  $\gamma$  is a stopping time). As a consequence, we give a full description of all local martingales and sigma-martingales in this model and classify them according to their global behaviour.

**Said Hamadem, Le Mans University, France.**

Mean-field reflected backward stochastic differential equations.

In this talk, we study a class of reflected backward stochastic differential equations (BSDEs) of mean-field type, where the mean-field interaction in terms of the expected value  $E[Y]$  of the  $Y$ -component of the solution enters both the driver and the lower obstacle. We consider the case where the lower obstacle is a deterministic function of  $(Y, E[Y])$ . Under mild Lipschitz and integrability conditions on the coefficients, we obtain the well-posedness of such a class of equations. Under further monotonicity conditions we show convergence of the standard penalization scheme to the solution of the equation. This class of models is motivated by applications in pricing life insurance contracts with surrender options.

**Andreas Hamel, Free University of Bozen-Bolzano, Italy.**

Multi-utility maximization: why, when and how?

The central question of the talk is the following: how can one solve a (very general) multi-utility maximization problem, i.e., find best alternatives w.r.t. a non-complete preference relation which has a multi-utility representation? This question will be reframed as a complete lattice-valued optimization problem. Moreover, motivations are given, several issues with naive approaches discussed and solution concepts introduced for the resulting problems with set-valued objectives. In the rest part, it is shown that many order relations defined or characterized in terms of families of scalar functions generate a complete lattice of sets (and a corresponding extension of the order relation) which turns the problem of finding best alternatives w.r.t. the order relation into a complete lattice-valued optimization problem. Examples are multi-utility representations due to Evren, Ok and others, stochastic dominance orders for the uni- and multivariate case, Bewley preferences as well as risk preferences for markets with transaction costs (and many more). In the second part, solution concepts are introduced and a few building blocks for a corresponding multi-utility maximization theory are provided, in particular a duality approach. In the third part, a few ramifications are discussed such as relationships to no-arbitrage concepts in markets with transaction costs.

**Philipp Harms, Freiburg University, Germany.**

Term structure modelling using cylindrical measure-valued processes.

Financial markets may experience shocks at predictable times, for instance due to votes, quarterly reports, dividend payments, or regulatory constraints. This creates discontinuities in the term structure of forward prices and forces the instantaneous forward rates to be measure-valued. Unfortunately, stochastic analysis for measure-valued processes meets severe technical difficulties. However, these difficulties can be circumvented using cylindrical measure-valued processes, which are both easier to handle and sufficient for financial applications. As an example, I will show how to construct joint models for energy and interest rate products in this framework.

**Emma Hubert, Paris-Est Marne-la-Vallée University, France.**

Hierarchical principal-agent problems.

This paper studies continuous-time optimal contracting in a hierarchy model which generalizes the model of Sung (2015). The hierarchy is modeled by a series of interlinked Principal-Agent problems, leading to a sequence of Stackelberg equilibria. More precisely, the Principal can contract with the Managers to incentivise them to act in her best interest, despite only observing the net benefits of the total hierarchy. Managers in turn subcontracts the Agents below them. Both Agents and Managers each independently control a stochastic process representing their outcome. First, we show through a simple example that even if the Agents only control the drift of their outcome, the Managers control the volatility of the Agents' continuation utility. Even this first simple example justifies the use of recent results on optimal contracting for drift and volatility control, and therefore the theory on 2BSDEs developed in the second more theoretical part of this paper, dedicated to a more general model with drift and volatility control. The comprehensive approach we outline opens the way to obtain general comparative statics. We also explain how our model can be extended to a large-scale principal-agent hierarchy. Since the Principal's problem can be reduced to only a  $2m$ -dimensional state space and a 2-dimensional control set, where  $m$  is the number of Managers just below her, and is therefore independent of the size of the hierarchy under these top Managers, the dimension of the problem does not explode.

**Alex Kalinin, LMU Munich, Germany.**

Support characterization for regular path-dependent stochastic Volterra integral equations.

We consider a stochastic Volterra integral equation with regular path-dependent coefficients and a Brownian motion as integrator in a multidimensional setting.

Under an imposed absolute continuity condition, the unique solution is a semi-martingale that admits almost surely Hölder continuous paths. Based on functional Itô calculus, we prove that the support of its law in the Hölder norm can be described by a flow of mild solutions to ordinary integro-differential equations that are constructed by means of the vertical derivative of the diffusion coefficient.

**Wahid Khosrawi-Sardroudi, ETH Zurich, Switzerland.**

Special session: Efficient computation of confidence bounds for neural network regression.

A prime example of machine learning is given by the supervised learning problem. Mathematically, this corresponds to a high dimensional non-linear regression problem, where the unknown function is approximated by a neural network and the goal is to find reasonable parameters for this network.

In application, an important question is about the level of certainty one has for the estimated function evaluated at a given point of the domain. We present an efficient approximating method to compute such confidence regions and analyze the performance by means of an appropriate test.

**Patrice Kiener, InModelia, France.**

Special session: On neural networks of perceptron type.

In this talk, I present some common and uncommon aspects of neural networks of perceptron type: a neuron, a neural network, static NN, static NN for mixtures (portfolio), dynamic NN for univariate and multivariate time series, the good and the bad training algorithms, direct calculation or back propagation for the Jacobian matrix, design of experiments applied to static NN, commercial and open source software, 12 good and 37 bad packages/functions in R. Finally, I will underline the differences between these neural networks and the recently introduced deep neural networks used in pattern recognition.

**Matti Kiiski, University of Mannheim, Germany.**

Pathwise hedging-pricing duality on the Skorokhod space.

We derive a dual representation for the minimal "model-free" superhedging cost of pathwise options on the Skorokhod space. The class of hedges used in superhedging needs to be carefully chosen depending on the pathwise properties of the payoff function. In particular, by allowing dynamic trading with countably many simple strategies with a common credit line turns out to be sufficient for the continuity of the minimal superhedging cost. We link this continuity to the compactness of the respective pricing models, and highlight its role in extensions of the duality correspondence for more complicated derivative contracts.

Our approach is based on the convex conjugate duality and pathwise hedging arguments.

**Nino Kordzakhia, Macquarie University, Australia.**

First passage time distributions for Brownian motion via integral equations.

The integral equations approach to the evaluation of first passage time distributions will be discussed for different classes of transformations of Brownian motion and time-dependent boundaries. The applications include the distributions of modified Kolmogorov-Smirnov type statistics and pricing of barrier options. The results of numerical experiments have been validated using alternative approaches and MC simulation method. Joint work with A. Novikov.

**Anastasis Kratsios, ETH Zurich, Switzerland.**

Special session: Deep learning in a generalized HJM-type framework through arbitrage-free regularization.

We introduce a regularization approach to arbitrage-free factor-model selection. The considered model selection problem seeks to learn the closest arbitrage-free HJM-type model to any prespecified factor-model. An asymptotic solution to this, a priori computationally intractable, the problem is represented as the limit of a 1-parameter family of optimizers to computationally tractable model selection tasks. Each of these simplified model-selection tasks seeks to learn the most similar model, to the prescribed factor-model, subject to a penalty detecting when the reference measure is a local martingale-measure for the entire under-lying financial market. A simple expression for the penalty terms is obtained in the bond market withing the affine-term structure setting, and it is used to formulate a deep-learning approach to arbitrage-free affine term-structure modelling. Numerical implementations are also performed to evaluate the performance in the bond market.

**Christoph Kuhn, University of Frankfurt, Germany.**

Semimartingale price systems in models with transaction costs beyond efficient friction.

Standing assumptions in the literature on proportional transaction costs are efficient friction and robust no-arbitrage that rule out strategies of infinite variation, as they usually appear in frictionless markets. In this talk, we show how the models with and without transaction costs can be unified.

The bid and the ask price of a risky asset are given by càdlàg processes which are locally bounded from below and may coincide at some points. In a first step, we show that if the bid-ask model satisfies “no unbounded profit with bounded

risk” for simple strategies, then there exists a semimartingale lying between the bid and the ask price process. In a second step, under the additional assumption that the zeros of the bid-ask spread are either starting points of an excursion away from zero or inner points from the right, we show that for every bounded predictable strategy specifying the amount of risky assets, the semimartingale can be used to construct the corresponding self-financing risk-free position in a consistent way. The talk is based on joint work with Alexander Molitor.

**Yuri Kutoyants, Le Mans University, France.**

Statistical inference for SDE with delays.

We present a review of several results of statistical estimation for stochastic differential equations with delay. The delay estimation is discussed in two asymptotics: small noise and large samples. We propose One-step MLE of delay and describe its asymptotic behavior. Nonparametric estimation problems are considered in small noise case only. In all problems we discuss the optimality of the proposed estimators.

**Houzhi Li, Paris Diderot University, France.**

Modeling the market by weights.

We study the generalization of a stochastic differential equation arising from nonlinear optimal filtering problem. This equation has some interesting properties, for example it has a unique strong solution that remains in the unit simplex, that we use for the modeling of relative weights of assets in a capital market. Furthermore, with some additional assumptions, we recognize the some well-known market models in stochastic portfolio theory such as volatility-stabilized markets. Based on this model of market weights, we investigate long-term behavior of the market, relative arbitrage opportunities and other portfolios generated by this market structure. This model can be applied to stock market indices, where the weights correspond to relative market prices or capitalizations of stocks.

**Andrew Lyasoff, Boston University, USA.**

Incomplete-market equilibria with a large number of heterogeneous households and their connection with discrete mean field games and control.

Incomplete-market equilibrium models with a very large number of heterogeneous agents do parallel models of mean field games and control, but also involve interactions that cannot be placed in the standard mean field framework. Specifically, while the cross-sectional distribution of the agents’ wealth is critical, the agents must still coordinate their individual choices of the control variables (consumption and portfolio choices) because: (a) they must agree

on the (endogenous) asset prices, and (b) their choices must clear the market. In fact, as the prices are endogenous, the individual control problems are not even meaningful until all agents agree on those prices. Nevertheless, by using the technique introduced by Dumas and Lyasoff (JoF, 67:5) it becomes possible to solve some common equilibrium models of incomplete markets with a large number of heterogeneous agents (so to speak, to solve mean field games with "orchestrated" strategies). This technique is based on a special decoupling of the first order conditions, organized in such a way that at every step one solves simultaneously for certain control variables attached to the current period and other control variables attached to the next period. There is a very close connection between these steps and the principle of maximum formulated in terms of BSDEs. The main difference in relation to (JoF, 67:5), which assumes a small number of agents, is in the need to account for the cross-sectional distribution of agents (whence the parallel with the mean field terminology and methodology). It will be shown how the proposed new approach can be applied to classical mean field control problems in discrete time, potentially leading to new approximation methods. The talk will conclude with concrete examples of Bewley-Aiyagari models taken from classical macroeconomic texts and will discuss the difference between the proposed new approach and some common solution methods. Specifically, it will be shown that the classical approach (see Ch. 18 in Ljungqvist and Sargent, for example) to such models fails to produce an equilibrium – even in the very examples that are meant to illustrate the classical approach.

**Gennady Martynov, IITP RAS, Moscow, Russia.**

One method for using of the Cramér-von Mises test with estimated parameters.

The limit distribution of the classical Cramèr-von Mises statistic with unknown estimated parameters depends from these parameters. It is well known the strong Khmaladze method for overcoming these difficulties. But its practical using requires complex transformations that must be done every time a hypothesis is tested. It is known only one example application of this method to a parametric distribution family. This is the exponential family. We can note that for the most using on the practice distribution families the dependence from parameters is missing within families. Corresponding tables exist for the statistics percentiles. However, in the general case, we can use its estimator as an approximation of the unknown vector of parameters. The error in determining the threshold level will be insignificant for a sufficiently large number of observations. Examples are given.

**Jean-Marc Mercier, MPG.Partners, France.**

The transport-based meshfree method (TMM) and its applications in finance:  
a review.

We review a numerical technique, referred to as the Transport-based Mesh-free Method (TMM), and we discuss its applications to mathematical finance. We recently introduced this method from a numerical standpoint and investigated the accuracy of integration formulas based on the Monte-Carlo methodology: quantitative error bounds were discussed and we outline the main ideas of our approach. The techniques of transportation and reproducing kernels lead us to a very efficient methodology for numerical simulations in many practical applications, and provide some light on the methods used by the artificial intelligence community. For applications in the finance industry, our method allows us to compute many types of risk measures with an accurate and fast algorithm. We propose theoretical arguments as well as extensive numerical tests in order to justify sharp convergence rates, leading to rather optimal computational times. Cases of direct interest in finance support our claims and the importance of the problem of the curse of dimensionality in finance applications is briefly discussed.

**Enzo Miller, Paris 7 University, France.**

Linear-quadratic control problems for a class of stochastic Volterra equations of convolution type.

We provide an exhaustive treatment of Linear-Quadratic control problems for a class of stochastic Volterra equations of convolution type, whose kernels are Laplace transforms of certain signed matrix measures which are not necessarily finite. These equations are in general neither Markovian nor semimartingales, and include the fractional Brownian motion with Hurst index smaller than  $1/2$  as a special case. We establish the correspondence of the initial problem with a possibly infinite dimensional Markovian one in a Banach space, which allows us to identify the Markovian controlled state variables. Using a refined martingale verification argument combined with a squares completion technique, we prove that the value function is of linear quadratic form in these state variables with a linear optimal feedback control, depending on non-standard Banach space valued Riccati equations. Furthermore, we show that the value function of the stochastic Volterra optimization problem can be approximated by that of conventional finite dimensional Markovian Linear-Quadratic problems, which is of crucial importance for numerical implementation.

**Ludger Overbeck, University of Gießen, Germany.**

Systemic risk and capital allocation.

In this talk we extend the axiomatic approach to systemic risk, as introduced in Chen et al. (2013), in different directions. One direction is the introduction of systemic risk measures that do not have to be positively homogeneous. The other direction is that we allow for a general measurable space. This extends the scope of possible loss distributions of the components of a financial system to a

great extent and introduces more flexibility for the choice of suitable systemic risk measures. In the second part we review and discuss different approach to capital allocation, which might be related to different settings of systemic risk measures.

**Ilya Pavlyukevich, Friedrich Schiller University Jena, Germany.**

Non-Gaussian limit theorem for non-linear Langevin equations driven by Lévy noise.

We consider a second-order Langevin equation for the motion of a particle subject to a non-linear friction force being a power of the particle's velocity,  $F = -|v|^\beta \text{sign}(v)$ ,  $\beta \in \mathbb{R}$ , and random vibrations. We determine the law of the displacement process in the limit of the small noise amplitude. This is the joint work with A. Kulik (TU Wroclaw).

**Teemu Pennanen, King's College London, England.**

Special session: Information-based complexity in stochastic optimization.

Information based complexity (IBC) studies the limits of solution accuracy obtainable with a given amount of information. IBC has been extensively studied in the context of numerical integration and optimization. We study IBC of stochastic optimization problems that are fundamental e.g. in financial mathematics and machine learning. Combining existing results on integration and optimization, we identify classes of convex stochastic optimization problems whose IBC is polynomial in the problem dimensions. Although simple, this seems to be the first tractability result in stochastic optimization.

**Dylan Possamai, Colombia University, USA.**

Contract theory for time-inconsistent agents.

This talk is a follow-up on the one I gave last year. We consider a general non-Markovian theory of time-inconsistent stochastic control problems in continuous-time where a sophisticated agent, who is aware of his time-inconsistency, takes into account in future decisions. We show how the characterisation we obtain can be applied to contract theory problems, highlighting in particular a potential fruitful link with BSVEs. This is a joint work with Camilo Hernández, Columbia University.

**Thorsten Schmidt, Friedrich Schiller University Jena, Germany.**

No arbitrage in insurance.

This paper is an attempt to define and link absence of arbitrage and valuation of insurance contracts by using the law of large numbers and an extension to insurance events of risk neutral valuation of financial markets. It is joint work with Philippe Artzner and Karl-Theodor Eisele.

**Makoto Shimoshimizu, Osaka University, Japan.**

Optimal execution strategies with generalized price impacts in a continuous-time setting.

In this paper, we analyze a continuous-time analog of the optimal trade execution problem with generalized price impacts, which was recently discussed in Ohnishi and Shimoshimizu (2019) for a discrete-time setting. The market model considers transient price impacts of random trade execution volumes posed by small traders as well as a large trader. Our problem is formulated as a stochastic continuous control problem over a finite horizon of maximizing the expected utility from the final wealth of the large trader with Constant Absolute Risk Aversion (CARA) von Neumann-Morgenstern (vN-M) utility function. By examining the Hamilton-Jacobi-Bellman (HJB) equation, we characterize the optimal value function and optimal trade execution strategy, and conclude that the trade execution strategy is a time-dependent affine function of three state variables: the remained trade execution volume of the large trader and, so-called, the residual effects of past price impacts caused by both of the large trader and other noise-traders and the noise traders' aggregate volume of order itself. Further, the time-dependent coefficients could be derived from a solution of a system of Ordinary Differential Equations (ODEs) with terminal conditions, which is numerically tractable.

**Laura Tinsi, Crest Ensaie/EDF, France.**

Price formation and optimal trading in intraday electricity markets.

We study price formation in intraday electricity markets in the presence of heterogeneous agents, asymmetric information and intermittent generation. We use stochastic control theory to identify optimal strategies of agents with market impact and exhibit the Nash equilibrium in closed form for a finite number of agents as well as in the asymptotic setting of Mean field games. We show that our model is able to reproduce some empirical facts observed in the market (price impact, volatility), and allows producers to deal with risks and costs related to intermittent renewable generation.

**Lioudmila Vostrikova, Larema, Angers University, France.**

Life insurance of the euro-denominated contracts.

The life insurance of the euro-denominated contracts is very popular investment in France, for example it represents more than 1600 billion at the end of 2016. But the interest rate of this financial product is no longer very satisfactory because it decreases very strongly : it was 4% in 2000 and only 1,8% in average in 2017. The decrease is logical : life insurers invest mostly in long-term Government bonds. The return of 10-year French Government bonds was about 5% in 2000 and less than 1% in 2017. To increase the interest rate, one can invest in more risky assets. But how one can control the risk ? Some models will be proposed and some possible solutions will be given.

**Mikhail Zhitlukhin, Steklov Mathematical Institute, Moscow, Federal Russian Republic.**

Growth optimal strategies in a game model of a market.

We consider a game model of a market where several players (investors) compete for distribution of payoffs yielded by several assets. For this model, we study strategies which are optimal on the infinite time horizon in the sense that they achieve the maximal rate of growth compared to the strategies of the competitors. Such a problem is analogous to the problem of finding an optimal growth strategy (numeraire) in a single-investor market, but the essential difference is that in our model the gain or loss of an investor depends also on actions of the other investors. The main result is a construction of an optimal strategy in an explicit form. Moreover, we show that it is unique in some sense. This talk will be mostly focused on a discrete-time model, although some preliminary results for a continuous-time model will be presented as well.

**Peng Wu, Paris-Dauphine University, PSL, France.**

Queue-reactive Hawkes model for the order flow.

Today, electronic trading has developed considerably compared to decades ago. Numerous studies have been done to better understand the order flow, as well as market participants behaviours. Among these studies, modelling the limit order book (LOB for short) has always played an essential role. Indeed, a successful modelling of the limit order book allows to capture microscopic behaviours. Furthermore, a LOB model enables us to simulate the dynamics of LOB, which is helpful to improve trading strategies and manage risks. Among the different models for the order flow, Hawkes point processes have proven to be a viable approach. In this work, we aim at combining the dependence on past order flow with dependence on the current state of the book. To do so, we present a Hawkes process based queue-reactive model. Joint work with Marcello Rambaldi, Jean-Francois Muzy, and Emmanuel Bacry.